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# MOTION ANALYSIS OF A TRAILED HARVESTER WITH COMBING WORKING UNITS

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ABSTRACT

The article presents the motion analysis of a trailed harvester equipped with combing working units. The harvesting unit comprises a wheeled tractor, a harvesting machine, and a trailer for collecting the combed heap. The research aims to model the behaviour of the machine under various simulation conditions. To facilitate the analysis of the motion of the three-link harvesting aggregate, constraints of the harvester to the tractor and trailer were replaced with their reactions, and the motion of a single harvester was considered. In the first stage of the studies, a calculation scheme was drawn up indicating the forces and moments of forces influencing the machine and the constraint reactions. Lagrange's equation of the second kind in generalised coordinates was used to derive a differential equation of the machine's motion. The rotation angle of the harvester relative to the hitch point with the tractor was taken as a generalised coordinate. After algebraic transformations, a differential equation for the harvester's motion was obtained. By solving the differential equation, a function was found, which made it possible to analyse the change in the rotation angle of the machine. Further analysis of the motion of the harvester was carried out using experimental methods.

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ABSTRACT

Experimental data is used to verify the model's accuracy and its correspondence to real processes. If the model accurately predicts the behaviour of the combed heap, it confirms its adequacy. The parameters of the mathematical model can be adjusted based on the results of experiments to improve the accuracy of the prediction. The mathematical model allows predicting the behaviour of the combed heap under various conditions, which can help optimise the process parameters. Experiments can produce results that are difficult to interpret without a theoretical framework. The model helps to explain the mechanisms underlying the observed phenomena. The programme of experimental studies included obtaining the statistical characteristics of the horizontal oscillation amplitude of the harvester within the speed range of 1.2-2.8 $m \cdot s^{-1}$. The minimum deviation of the mass centre of the harvester from its linear motion was adopted as an estimation criterion for its linear motion. As a result of experimental studies which will be presented in the next article, it was determined that the most acceptable mode of motion that meets this requirement and provides the maximum efficiency of the harvester is the motion speed of 2.0 $m \cdot s^{-1}$.

## Introduction

One approach to solving the problem of efficient cereal harvesting is to introduce the method of crop cultivation by combing on the stalk. In Melitopol Institute of Agricultural Mechanization (currently Dmytro Motornyi Tavria State Agrotechnological University), under the guidance of Professor P.A. Shabanov, research was carried out to substantiate the design parameters and operation modes of combing devices, which can be used for combine and non-combine harvesting.

As proved in (Lezhenkin et al., 2021; Tabor et al., 2019; Halko et al., 2023), the non-combine harvesting method is the most effective option, which involves collecting the combed grain heap using a harvester and then separating it under stationary conditions. The separation of the combed grain heap is further elaborated in research papers (Zhang et al., 2022; Margielewicz et al., 2023; Liu et al., 2023; Bazaluk et al., 2022a).

A trailed harvester with combing working units for harvesting the combed grain heap was developed at Tavria State Agrotechnological University. For over three decades, scientists at the university have conducted research to justify the structural and technological parameters of the combing device.

However, no studies on the motion dynamics, which aim to substantiate the stable motion of a trailed harvester with combing working units, have been carried out.

Development of a dynamic model of the plane-parallel motion of a tractor field machine in the horizontal plane is described in works (Havrylenko et al., 2021a; Havrylenko et al., 2021b). In these papers, theoretical studies are given, which are based on the provisions of statistical dynamics of agricultural aggregates, the theory of mobile power facilities, theoretical mechanics, and the theory of automatic control of linear dynamic systems with statically random control of perturbing input effects. The study of the smoothness of a tractor and a vehicle with a trailer is described in (Karaiev et al., 2021). The authors used Lyapunov's stability theory to study the dynamics of a vehicle with a trailer.

Lyapunov's method is used to determine the stability of tractor motion when the traction force changes (Sun and He, 2018). The parameters are evaluated by collecting the experimental data. The study of the stability of a semi-trailer tractor is presented in (Zhao et al., 2019; Ding et al., 2014), in which the stability of motion is analysed using the stability index.

Article (Ko and Lee, 2002) substantiates the method of determining vehicle stability using the topological approach. An algorithm based on the trajectory method is proposed to determine the exact stability area. Numerical analysis is carried out using a nonlinear vehicle model.

Paper (Myasishchev et al., 2020) deals with the motion study of the forestry tractor using statistical dynamics. External random effects of the surface micro profile on which the tractor moves are adopted as the input variables, and the hourly fuel consumption is the output process. This model enabled the optimisation problem to be solved by choosing the optimum tyre size, providing minimum energy consumption (Bazaluk et al., 2022b; Bazaluk et al., 2022c).

The authors of (Bulgakov et al., 2020) consider the motion of the machine section for black fallow cultivation. The article presents a mathematical model that describes the motion dynamics of the harrow's section in the longitudinal vertical plane and gives its solution which allows research on the influence of the design parameters on the rotation angle of the harrow's section and the time it needs to reach the state of equilibrium.

Theoretical studies on the front-mounted machine aggregation are considered in the article (McNabb and Startsev, 2022) which provides a methodology for determination of the optimal parameters of beet harvesting aggregates according to efficiency and energy consumption criteria.

Academician V.P. Goryachkin laid the foundation of the motion theory of agricultural machines and aggregates. He emphasised the importance of establishing the correct relationships between the forces acting on the machine and their masses, speeds, and operating modes. He proposed a theory of masses and speeds of machines, which is related to the research on their motion dynamics and stability. Further research on the dynamics of agricultural aggregates was reflected in the research work of L.M. Vasylenko. As applied to agricultural traction units, the most complete research on the motion dynamics and stability is provided in works (Ko and Lee 2002; Myasishchev et al., 2020), where the trailer is considered as a physical pendulum, which is in relative motion along the horizontal surface under the action of the resistive forces of the working units, taking into account the reactions of the carrier wheel tyres.

The problem of grain harvesting by asymmetric machine-tractor aggregates is addressed in work, which considers the optimisation of the technical and economic indicators from the point of view of the motion stability of the reaper in the horizontal plane.

System identification of the new yaw pattern of an agricultural tractor is the research subject of (Bevly et al., 2002; Dzieniszewski et.al. 2023). This paper presents the dynamic model of tractor yaw, along with its analysis and experimental data.

However, no analysis of the motion of the trailed grain harvester with combing working units has been carried out previously. This poses the challenge of investigating the rational modes of its motion while harvesting grain crops.

## Materials and Methods

To carry out the research, a programme has been developed consisting of the following stages:

- drawing up a calculation scheme of the harvester indicating the forces and the moments of forces and the constraint reactions;
- building a mathematical model of the relative motion of the harvester in the form of a differential equation using the calculation scheme and Lagrange's equation of the second kind in generalised coordinates;
- solving the differential equation with respect to the generalised coordinate $\varphi_2$;
- investigation of the solution obtained;
- carrying out the experimental research of the motion of the harvester as a component of a three-link harvesting aggregate.

The article focuses on the results of theoretical studies, which is a preparatory stage for comparison with experimental studies. The results of the comparison will be described in detail in our next article.

The theoretical research methodology is based on the provisions of theoretical mechanics, highlighted in (Bulgakov et al., 2022), and the algorithm for deriving differential equations presented in (Zhang et al., 2022). Methods of further mathematics were used to solve the differential equations (Rubtsov et al., 2015; Rubtsov et al., 2019).

The methodology of the experimental studies was as follows. Before the test runs, the selected plot of the field was divided into test sections of 50 meters each. For acceleration and exit of the aggregate from the plot, additional areas of 10 meters each were allocated.

The experiments were carried out as follows: on command, the aggregate passed the test plot at a certain gear with a fixed forward speed. The forward speed of the aggregate was determined from the ratio:

$$V = \frac{l_p}{t} \tag{1}$$

where:

$l_p$ – length of the test plot, $l_p = 50$ m;
$t$ – plot clearance time, s.

The trajectory of the relative motion of the harvester was recorded using a marker permanently attached at the corresponding point. The minimum values of the statistical characteristics of the amplitudes of horizontal oscillations of the harvester's centre of mass were adopted as the evaluation criterion for the stable motion zone.

To obtain numerical estimates of the harvester's centre of mass disconnections relative to the fixed axis, they were measured in steps of $\Delta l = 1$ m, each test consisted of 50 values.

As a result of processing the primary information obtained in the experiment, estimations of the statistical characteristics of the amplitudes of horizontal oscillations of the harvester's centre of mass were performed at six speeds of motion within the range of 1.2-2.87 $m \cdot s^{-1}$.

## Results and Discussion

Let us draw up a calculation scheme for the harvester with specified vectors of active forces and moments of forces (Fig. 1).

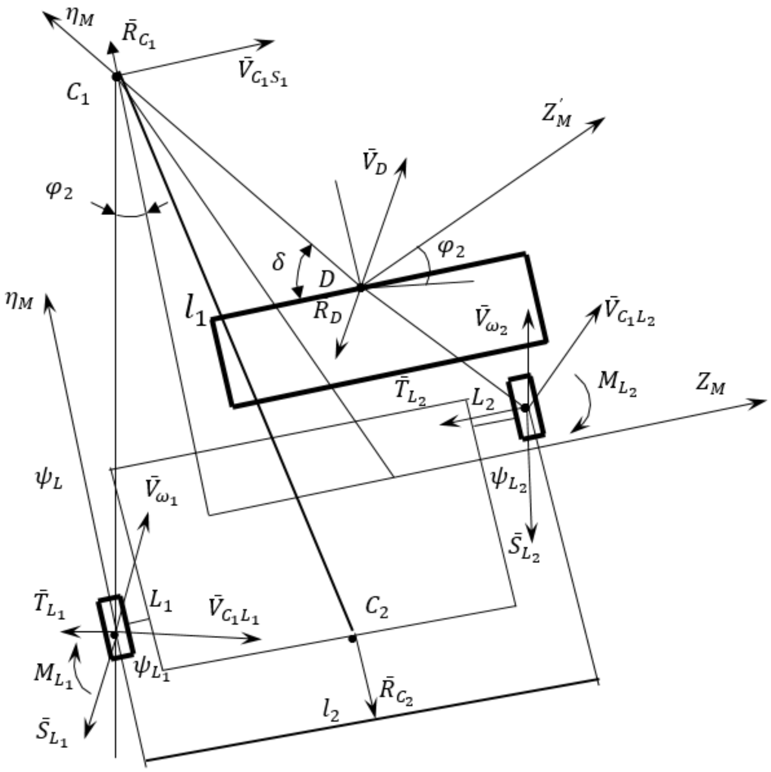


*Figure 1. Calculation scheme of the harvester*

In Figure 1, the following designations are used: $\bar{T}_{L_1}$ and $\bar{T}_{L_2}$ – elastic forces of tyres of left and right wheels of the harvester, $M_{L_1}$ and $M_{L_2}$ – moments of elastic forces of tyres of left and right wheels of the harvester, $\bar{S}_{L_1}$ and $\bar{S}_{L_2}$ – resistant forces of tyres of left and right wheels of the harvester, $\bar{R}_{C_1}$– constraint reaction with the tractor, $\bar{R}_{C_2}$ – constraint reactions with the trailer for collecting the combed heap, $\bar{R}_D$ – main vector of resistant forces of combing.

In the first approximation, let us consider the elastic forces, their moments, as well as the resistant forces of the tyres of the left and right wheels of the harvester equal, that is, $\bar{T}_{L_1} \approx \bar{T}_{L_2} \approx \bar{T}_L$, $M_{L_1} \approx M_{L_2} \approx M_L$ and $S_{L_1} \approx S_{L_2} \approx S_L$ .

To build a differential equation of the harvester's motion we use Lagrange's equation of the second kind in generalized coordinates (Bulgakov et al., 2022). The generalized coordinate is the rotation angle $\varphi_2$ of the harvesting machine relative to the trailer's hitch point with the tractor.

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\varphi}_2}\right] - \frac{\partial T}{\partial \varphi_2} = Q \tag{2}$$

where:
$T$ – kinetic energy of the harvester in relative motion, J
$\dot{\varphi}_2$ – generalized speed, $\left(\frac{d\varphi_2}{dt}\right)$, $s^{-1}$
$Q$ – generalized force, N·m

The differential equation for the relative motion of the harvester is as follows:

$$I_{C_1} \cdot \ddot{\varphi}_2 = -T_L \cdot l - T_L \cdot l \cdot \sqrt{1 - \frac{(l_1)^2}{l^2}} - 2M_L - S_L \cdot l \cdot \psi_L - S_L \cdot l \cdot \sqrt{1 - \frac{(l_1)^2}{l^2}} \cdot \psi_L - \\ - R_D \cdot L_R \cdot \gamma_M - R_{C_2} \cdot \varphi_2 \cdot l_1 \tag{3}$$

where:
$I_{C_1}$ – moment of inertia of the harvester relative to the axis passing through point $C_1$ of the trailer to the tractor, kg·m$^2$
$\ddot{\varphi}_2$ – angular acceleration of the harvester in relative motion, $s^{-2}$
$l$ – distance from the hitch point of the harvester $C_1$ to its wheels, m
$\psi_L$ – tyre twist angle of the harvester's wheels, rad
$L_R$ – distance from the hitch point of the harvester $C_1$ to the application point of the main vector of the resistive force $D$, m
$\gamma_M$ – angle made by the speed vector of point $D$ with axis $O_1Y_1$, rad
$l_1$ – distance from the hitch point $C_1$ connecting the harvester to the tractor to the hitch point $C_2$ connecting the trailer to the harvester;
$l_2$ – distance between the centre of the left wheel $L_1$ and the right wheel $L_2$ of the harvester.

Equation (3) includes the elasticity force of the tyres $T_L$, the moment of the elasticity force of the tyres $M_L$ and the tyre twist angle of the harvester's wheels $\psi_L$. By substituting the expressions for their finding into Equation (3), we obtain:

$$I_{C_1} \cdot \ddot{\varphi}_2 = -C_L \cdot l \cdot \Delta_L - C_L \cdot l \cdot \sqrt{1 - \frac{p^2}{l^2}} \cdot \Delta_L - 2 \cdot k_L \cdot f_L \cdot \Delta_L - S_L \cdot l \cdot k_L \cdot \Delta_L - \\ - S_L \cdot l \cdot k_L \cdot \Delta_L \cdot \sqrt{1 - \frac{p^2}{l^2}} - R_D \cdot L_R \cdot \gamma_M - R_{C_2} \cdot \varphi_2 \cdot l_1 \tag{4}$$

where:
$\Delta_L$ – lateral strain of the tyres, *m*
$f_L$ – tyre twist stiffness coefficient, *N·m*

$k_L$ – proportionality coefficient, $m^{-1}$
$C_L$ – tyre shift stiffness coefficient, $\frac{N}{m}$

Let us introduce the following designations:

$$L = -C_L \cdot l - C_L \cdot l \cdot \sqrt{1 - \frac{p^2}{l^2}} - 2 \cdot k_L \cdot f_L - S_L \cdot l \cdot k_L - S_L \cdot l \cdot k_L \cdot \sqrt{1 - \frac{p^2}{l^2}}. \tag{5}$$

As a result, we obtain:

$$I_{C_1} \cdot \ddot{\varphi}_2 + R_D \cdot L_R \cdot \gamma_M + R_{C_2} \cdot \varphi_2 \cdot l_1 = \Delta_L \cdot L \tag{6}$$

Let us solve Equation (6) with respect to $\Delta_L$:

$$\Delta_L = \frac{I_{C_1} \cdot \ddot{\varphi}_2 + R_D \cdot L_R \cdot \gamma_M + R_{C_2} \cdot \varphi_2 \cdot l_1}{L} \tag{7}$$

Let us substitute the value of the angle $\gamma_M$, obtained in (Bulgakov et al., 2010) into Equation (7):

$$\Delta_L = \frac{I_{C_1} \cdot \ddot{\varphi}_2 + \frac{R_D \cdot L_R}{V_0}(V_0 \cdot \varphi_2 + \dot{\varphi}_2 \cdot C_R) + R_{C_2} \cdot \varphi_2 \cdot l_1}{L} \tag{8}$$

Let us differentiate Equation (8) with respect to time:

$$\frac{d\Delta_L}{dt} = \frac{I_{C_1} \cdot \dddot{\varphi}_2 + \frac{R_D \cdot L_R}{V_0}(V_0 \cdot \ddot{\varphi}_2 + \dot{\varphi}_2 \cdot C_R) + R_{C_2} \cdot \dot{\varphi}_2 \cdot l_1}{L} \tag{9}$$

At the same time, based on the formula given (Bulgakov et al., 2010),

$$\frac{d\Delta_L}{dt} = -V_0 \cdot k_L \cdot \Delta_L + V_0 \cdot \varphi_2 + \dot{\varphi}_2 \cdot l \tag{10}$$

Let us equate the right parts of Equations (9) and (10):

$$\frac{I_{C_1} \cdot \dddot{\varphi}_2 + \frac{R_D \cdot L_R}{V_0}(V_0 \cdot \ddot{\varphi}_2 + \dot{\varphi}_2 \cdot L_R) + R_{C_2} \cdot \dot{\varphi}_2 \cdot l_1}{L} = -V_0 \cdot k_L \cdot \Delta_L + V_0 \cdot \varphi_2 + \dot{\varphi}_2 \cdot l_1 \tag{11}$$

Let us substitute the tyres strain values the harvester's wheels $\Delta_L$ (7) into Equation (11) and transform the obtained equality:

$$I_{C_1} \cdot \dddot{\varphi}_2 + \frac{R_D \cdot C_R^2}{V_0} \ddot{\varphi}_2 + R_{C_2} \cdot \dot{\varphi}_2 \cdot l_1 + R_D \cdot C_R \cdot \dot{\varphi}_2 = V_0 \cdot \varphi_2 \cdot L + \dot{\varphi}_2 \cdot l \cdot L -$$
$$-V_0 \cdot k_L \cdot I_{C_1} \cdot \ddot{\varphi}_2 + V_0 \cdot k_L \cdot R_{C_2} \cdot \varphi_2 \cdot l_1 - R_D \cdot C_R \cdot V_0 \cdot k_L \cdot \varphi_2 - R_D \cdot C_R^2 \cdot k_L \cdot \dot{\varphi}_2 \tag{12}$$

Finally, we shall obtain the differential equation of the following form:

$$C_0 \cdot \dddot{\varphi}_2 + C_1 \cdot \ddot{\varphi}_2 + C_2 \cdot \dot{\varphi}_2 + C_3 \cdot \varphi_2 = 0 \tag{13}$$

where:

$$C_0 = I_{C_1} \tag{14}$$

$$C_1 = \frac{R_D \cdot L_R^2}{V_0} + V_0 \cdot k_L \cdot I_{C_1}$$

$$C_2 = R_D \cdot L_R + R_{C_2} \cdot L + R_D \cdot L_R^2 \cdot k_L$$

$$C_3 = -V_0 \cdot L + V_0 \cdot k_L \cdot R_{C_2} \cdot l_1 + R_D \cdot L_R \cdot V_0 \cdot k_L$$

Equations (14) are the coefficients of the differential Equation (13). To solve the resulting equation, let us bring the obtained equation to the form in which the coefficient in the leading derivative is zero. For this purpose, let us divide Equation (13) by $C_0$.

$$\dddot{\varphi}_2 + \frac{C_1}{C_0}\ddot{\varphi}_2 + \frac{C_2}{C_0}\dot{\varphi}_2 + \frac{C_3}{C_0}\varphi_2 = 0 \qquad (15)$$

For Equations (14), let us derive a characteristic equation:

$$\lambda^3 + \frac{C_1}{C_0}\lambda^2 + \frac{C_2}{C_0}\lambda + \frac{C_3}{C_0} = 0 \qquad (16)$$

To find the roots of the algebraic equation of the third degree, let us use the method set out in (Rubtsov et al., 2015). First, by substituting

$$\lambda = x - \frac{C_1}{3C_0} \qquad (17)$$

we shall eliminate the term that includes $\lambda^2$:

$$\left(x - \frac{C_1}{3C_0}\right)^3 + \frac{C_1}{C_0}\left(x - \frac{C_1}{3C_0}\right)^2 + \frac{C_2}{C_0}\left(x - \frac{C_1}{3C_0}\right) + \frac{C_3}{C_0} = 0 \qquad (18)$$

After algebraic transformations, Equation (18) takes the form of

$$x^3 + \left(\frac{C_2}{C_0} - \frac{C_1^2}{3C_0^2}\right)x + \frac{2C_1^3}{27C_0^3} - \frac{C_1C_2}{3C_0^2} + \frac{C_3}{C_0} = 0 \qquad (19)$$

Let us introduce the designations of

$$p = \frac{C_2}{C_0} - \frac{C_1^2}{3C_0^2} \text{ and } q = \frac{2C_1^3}{27C_0^3} - \frac{C_1C_2}{3C_0^2} + \frac{C_3}{C_0} \qquad (20)$$

Taking into account the designations of (20), Equation (19) takes the following form:

$$x^3 + px + q = 0 \qquad (21)$$

By finding the roots of Equation (21), the roots of Equation (16) will be obtained, taking into account the designations of (17).

According to the fundamental theorem of algebra, Equation (21) has three complex roots. Let $x_0$ be one of these roots. Let us introduce the additional unknown and consider the polynomial

$$f(u) = u^2 - x_0 u - \frac{p}{3} \qquad (22)$$

Its coefficients are complex numbers and, therefore, have two complex roots, $\alpha$ and $\beta$, as follows from Vieta's formula,

$$\alpha + \beta = x_0 \tag{23}$$

$$\alpha\beta = -\frac{p}{3} \tag{24}$$

Let us substitute Equation (23) into Equation (21) and transform the resulting equality, we shall obtain

$$\alpha^3 + \beta^3 + (3\alpha\beta + p)(\alpha + \beta) + q = 0 \tag{25}$$

Expression $3\alpha\beta + p$ equals zero; as a result, Equation (25) takes the following form:

$$\alpha^3 + \beta^3 = -q \tag{26}$$

On the other hand, based on Equation (24), we have:

$$\alpha^3\beta^3 = -\frac{p^3}{27} \tag{27}$$

Equations (26) and (27) are the roots of the square equation with complex roots:

$$z^2 + qz - \frac{p^2}{27} = 0 \tag{28}$$

By solving (22), we shall obtain:

$$z = -\frac{q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

from which

$$\alpha = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}, \beta = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \tag{29}$$

Then, according to Cardano's formula,

$$x_0 = \alpha + \beta = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

Since the cube root radical equation has three values in the field of complex number, Equations (29) give three values for $\alpha$ and $\beta$.

The original Equation (21) has valid coefficients; therefore, according to recommendations given in (Rubtsov et al., 2015), the sign of the equation $\frac{q^2}{4} + \frac{p^3}{27}$ which stands under the square root sign in Cardano's formula will be crucial. Note that the sign of this equation is opposite to the sign of the discriminant

$$D = -4p^3 - 27q^2 = -108\left(\frac{q^2}{4} + \frac{p^3}{27}\right)$$

Then, similar to the quadratic equation, the discriminant can take three values depending on the sign. Let us do the research depending on the discriminant (Baek et al., 2022; Da Costa

Neto and Elorza, 2021; Leszczyński et al., 2021; Jobbágy et al., 2021; Jourgholami et al., 2021).

1. Suppose $D < 0$. In this case, in Cardano's formula, there is a positive number under the sign of each of the square radicals, and therefore, under the sign of each of the cubic radicals we have real numbers. However, the cubic root of a real number has one valid and two conjugate complex values. Thus:

$$\begin{cases} x_1 = \alpha_1 + \beta_1 \\ x_2 = -\frac{\alpha_1+\beta_1}{2} + i\sqrt{3}\frac{\alpha_1-\beta_1}{2} \\ x_3 = -\frac{\alpha_1+\beta_1}{2} - i\sqrt{3}\frac{\alpha_1-\beta_1}{2} \end{cases} \tag{30}$$

For our equation,

$$\alpha_1 = \sqrt[3]{-\frac{1}{2}\left(\frac{2C_1^3}{27C_0^3} - \frac{C_1C_2}{3C_0^2} + \frac{C_3}{C_0}\right) + \sqrt{\frac{1}{4}\left(\frac{2C_1^3}{27C_0^3} - \frac{C_1C_2}{3C_0^2} + \frac{C_3}{C_0}\right)^2 + \frac{1}{27}\left(\frac{C_2}{C_0} - \frac{C_1^2}{3C_0^2}\right)^3}} \tag{31}$$

$$\beta_1 = \sqrt[3]{-\frac{1}{2}\left(\frac{2C_1^3}{27C_0^3} - \frac{C_1C_2}{3C_0^2} + \frac{C_3}{C_0}\right) - \sqrt{\frac{1}{4}\left(\frac{2C_1^3}{27C_0^3} - \frac{C_1C_2}{3C_0^2} + \frac{C_3}{C_0}\right)^2 + \frac{1}{27}\left(\frac{C_2}{C_0} - \frac{C_1^2}{3C_0^2}\right)^3}} \tag{32}$$

However,

$$\begin{cases} \lambda_1 = x_1 - \frac{C_1}{3C_0} \\ \lambda_2 = x_2 - \frac{C_1}{3C_0} \\ \lambda_3 = x_3 - \frac{C_1}{3C_0} \end{cases} \text{or} \begin{cases} \lambda_1 = \alpha_1 + \beta_1 - \frac{C_1}{3C_0} \\ \lambda_2 = -\frac{\alpha_1+\beta_1}{2} - \frac{C_1}{3C_0} + i\sqrt{3}\frac{\alpha_1-\beta_1}{2} \\ \lambda_3 = -\frac{\alpha_1+\beta_1}{2} - \frac{C_1}{3C_0} - i\sqrt{3}\frac{\alpha_1-\beta_1}{2} \end{cases} \tag{33}$$

Taking into account (33), the solution of Equation (15) will be written in the form of:

$$\varphi_2 = A_1 e^{\left(\alpha_1+\beta_1 - \frac{C_1}{3C_0}\right)t} + A_2 e^{-\left(\frac{\alpha_1+\beta_1}{2} + \frac{C_1}{3C_0}\right)t} cos\sqrt{3}\frac{\alpha_1-\beta_1}{2}t + A_3 e^{-\left(\frac{\alpha_1+\beta_1}{2} + \frac{C_1}{3C_0}\right)t} sin\sqrt{3}\frac{\alpha_1-\beta_1}{2}t \tag{34}$$

where:

$A_1$, $A_2$, $A_3$ – arbitrary constants, and $\alpha_1$, $\beta_1$ are found from Equations (31) and (32).

$A_1$, $A_2$, $A_3$ can be defined by the initial conditions.

By analysing Equation (34), we can conclude that with $t \to \infty$, the last two products will shrink to zero, and in fact, we obtain an exponential dependence, the oscillations are damping.

2. Suppose $D = 0$ . Then $\alpha = \beta = \sqrt[3]{-\frac{q}{2}}$

Given $\alpha_1$ is the real value of the radical $\alpha$, then, from Equation (24), $\beta_1$ will also be a real number. Subsequently, in accordance with recommendations given in (Rubtsov et al., 2015), we obtain:

$$x_1 = 2\alpha_1, x_2 = -\alpha_1, x_3 = -\alpha_1$$

from which

$$\lambda_1 = 2\alpha_1 - \frac{C_1}{3C_0}, \lambda_2 = -\alpha_1 - \frac{C_1}{3C_0}, \lambda_3 = -\alpha_1 - \frac{C_1}{3C_0} \quad (35)$$

That is, if $D = 0$, then all the roots of Equation (21) are valid, and two of them are equal. Considering (35), the solution of Equation (15) can be written as follows:

$$\varphi_2 = A_1 e^{\left(2\alpha_1 - \frac{C_1}{3C_0}\right)t} + (A_2 + A_3 t)e^{-\left(\alpha_1 + \frac{C_1}{3C_0}\right)t} \quad (36)$$

where:

$A_1, A_2, A_3$ – arbitrary constants.

With $t \to \infty$, the second product, will shrink to zero, and again we get an exponential dependence.

3. Suppose $D > 0$.

In this case, there is a negative number under the sign of the inverse root of Cardano's formula, therefore, under the sign of cubic radicals we have conjugate complex numbers. Thus, all the values of the radicals are complex numbers. Among the roots of Equation (21), there must be at least one valid one. Let that be the root of $x_1 = \alpha_0 + \beta_0$.

Since both the sum of numbers $\alpha_0$ and $\beta_0$ and their product, which equals $-\frac{p}{3}$, are valid, these numbers are conjugate as the roots of the square equation with valid coefficients. In (Rubtsov et al., 2015) it was proved that all the three roots of Equation (21) will be valid and different, Cardano's formula is not used in this case. However, Equation (16) has three valid roots:

$$\lambda_1 = x_1 - \frac{C_1}{3C_0}, \lambda_2 = x_2 - \frac{C_1}{3C_0}, \lambda_3 = x_3 - \frac{C_1}{3C_0} \quad (37)$$

Considering Equation (37), the solution of Equation (15) can be written as:

$$\varphi_2 = A_1 e^{\left(x_1 - \frac{C_1}{3C_0}\right)t} + A_2 e^{\left(x_2 - \frac{C_1}{3C_0}\right)t} + A_3 e^{\left(x_3 - \frac{C_1}{3C_0}\right)t} \quad (38)$$

where:

$A_1, A_2, A_3$ – arbitrary constants.

In this case, we obtain a proper exponential dependence.

Thus, when the unit moves in the same mode, i.e., at a constant forward speed, the harvester produces damping oscillations in relative motion.

The next step was an experimental study of the effect of forward speed on the harvester's stability. The results of the experimental study will be presented in the next article, in which horizontal oscillation graphs for combed heaps with various working widths will be presented. Table 1 provides statistical characteristics of horizontal oscillations of the centre of mass of the harvester.

Table 1.
*Statistical characteristics of horizontal oscillations of the centre of mass of the harvester.*

| No. | | Speed of motion, (m·s$^{-1}$) | Mean value, ($10^{-2}$ m) | Mean square deviation, ($10^{-2}$ m) | Coefficient of variation, (%) | Statistical error | Confidence Interval: Lower boundary, ($10^{-2}$ m) | Confidence Interval: Upper boundary, ($10^{-2}$ m) |
|---|---|---|---|---|---|---|---|---|
| 1 | Idling | 1.200 | +4.702 | +0.522 | +11.093 | +0.073 | +4.578 | +4.825 |
| 2 | | 1.460 | +4.497 | +0.650 | +14.456 | +0.092 | +4.343 | +4.651 |
| 3 | | 1.800 | +5.333 | +1.081 | +20.272 | +0.153 | +5.077 | +5.589 |
| 4 | | 2.000 | +6.260 | +0.883 | +14.098 | +0.125 | +6.051 | +6.469 |
| 5 | | 2.500 | +10.370 | +1.846 | +17.798 | +0.261 | +9.933 | +10.807 |
| 6 | | 2.870 | +14.098 | +2.792 | +19.805 | +0.395 | +13.436 | +14.760 |
| 7 | Working stroke | 1.200 | +4.995 | +0.652 | +13.042 | +0.092 | +4.841 | +5.150 |
| 8 | | 1.460 | +5.399 | +0.712 | +13.179 | +0.101 | +5.230 | +5.567 |
| 9 | | 1.800 | +6.153 | +0.746 | +12.124 | +0.106 | +5.976 | +6.330 |
| 10 | | 2.000 | +6.430 | +0.780 | +13.132 | +0.110 | +6.245 | +6.615 |
| 11 | | 2.500 | +10.000 | +1.169 | +11.687 | +0.165 | +9.723 | +10.277 |
| 12 | | 2.870 | +14.808 | +1.704 | +11.509 | +0.241 | +14.404 | +15.212 |

As can be seen from Table 1, the average horizontal oscillation amplitude of the centre of mass of the harvesting machine (defined with a precision of up to three decimal places) is within the range of 4.7-14.1·$10^{-2}$, and when performing the technological process of harvesting, these are 4.995-14,808·$10^{-2}$ m. Consequently, the load does not affect the average horizontal oscillation amplitude of the harvester's centre of mass.

The mean square deviation of horizontal oscillations of the harvester's centre of mass at idling was 0.522-2.79·$10^{-2}$ m, and while harvesting it was 0.652-1.704·$10^{-2}$ m. The coefficient of variation at idling was 11.09-20.27%, and at a working stroke, it was 11.5-13.17%. The data show that the amplitude variations relative to the average values are smaller when the harvester moves at working stroke than when it moves at idle speed.

Moreover, with an increase in the speed of motion, the average amplitude of the harvester's centre of mass increases. Thus, at a speed of 2.0 m·s$^{-1}$, the average amplitude of the centre of mass is 6.43·$10^{-2}$ m, and at a speed of 2.87 m·s$^{-1}$ – 4.8·$10^{-2}$ m, i.e., the mean oscillation amplitude increased more than twice. This leads to an increase in grain losses harvested by using non-combing method.

Based on the above statistical analysis of the amplitude of horizontal oscillations of the harvester's centre of mass, it can be concluded that the most favourable mode of motion, which provides minimum deviations from the forward motion, as well as the maximum performance of the unit, is the speed of 2.0 m·s$^{-1}$.

## Conclusions

A mathematical model of relative motion of the harvester in the form of a second-order differential equation has been developed. The solution of the resulting differential equation and its subsequent analysis made it possible to establish that in relative motion the harvesting machine produces damping oscillations.

Experimental studies of the harvester's motion as part of a three-link harvesting aggregate have established that:

- the horizontal oscillation of the harvester is significantly affected by the speed of the aggregate's forward motion;
- the lowest values of amplitude (5.0-6.0) of the harvester's horizontal oscillation are observed within the range of forward motion speed of 1.2-2.0 $m \cdot s^{-1}$;
- with a further increase in the forward speed, there is a significant increase in the amplitude of oscillations up to $14.9 \cdot 10^{-2}$ m, which leads to an increase in grain losses when harvesting with the non-combing method.

The recommended speed of 2.0 $m \cdot s^{-1}$ ensures minimum grain loss and maximum efficiency.

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## ANALIZA RUCHU KOMBAJNU PRZYCZEPIANEGO Z CZESAJĄCYMI ELEMENTAMI ROBOCZYMI

**Streszczenie.** Niniejszy artykuł przedstawia analizę ruchu kombajnu przyczepianego wyposażonego w grzebieniowy zespół żniwny. Kombajn składa się z traktora, jednostki zbierającej oraz naczepy. Celem badania jest modelowanie zachowania maszyny w rożnych warunkach symulacji. Aby usprawnić analizę ruchu trzyczęściowego agregatu do zbioru, ograniczenia kombajnu wobec ciągnika i naczepy zastąpiono ich reakcjami i rozważono ruch pojedynczego kombajnu. W pierwszej fazie badań, przygotowano schemat obliczeniowy wskazujący na siły i momenty sił, które wpływają na maszynę i reakcje ograniczające. Zastosowano równanie Lagrange drugiego rodzaju w układzie współrzędnych uogólnionych, aby wyciągnąć równanie różniczkowe ruchu maszyny. Kąt obrotu jednostki zbierającej względny wobec punktu zaczepienia z ciągnikiem przyjęto jako uogólnioną współrzędną. Po transformacjach algebraicznych , otrzymano równanie różniczkowe dla ruchu kombajnu. Rozwiązując równanie różniczkowe, znaleziono funkcję, która umożliwiła analizę zmiany kąta obrotu maszyny. Dalsza analiza ruchu kombajnu została przeprowadzona za pomocą metod eksperymentalnych. Użyto danych eksperymentalnych do weryfikacji dokładności modelu i dopasowania do rzeczywistych procesów. Jeśli model dokładnie przewiduje zachowanie zebranego stosu, potwierdza to jego dokładność. Parametry modelu matematycznego pozwalają na przewidywanie zachowania zebranego stosu w różnych warunkach, co może pomóc w optymalizacji parametrów procesu. Doświadczenia mogą wpłynąć na wyniki, które trudno zinterpretować bez ram teoretycznych. Model pomaga wyjaśnić mechanizmy, które stoją za zaobserwowanymi zjawiskami. Program badań eksperymentalnych dotyczy uzyskania charakterystyki statystycznej poziomej amplitudy wahań kombajnu w zakresie prędkości 1,2-2,8 $m \cdot s^{-1}$. Minimalne odchylenie środka masy kombajnu od ruchu liniowego przyjęto jako kryterium oceny ruchu liniowego. W wyniku badań eksperymentalnych, które zostaną zaprezentowane w następnym artykule, określono akceptowalny tryb ruchu, który spełnia to wymaganie i zapewnia maksymalną wydajność kombajnu i wynosi 2,0 $m \cdot s^{-1}$.

**Słowa kluczowe:** kombajn, jednostki pracujące, równanie różniczkowe, stabilność ruchu, prędkość ruchu, siły, momenty sił