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OPTIMAL PROPERTIES OF INCOMPRESSIBLE SWIRLING FLOW IN A RECTILINEAR CIRCULAR PIPE

ОПТИМАЛЬНІ ВЛАСТИВОСТІ НЕСТИСЛИВОЇ ЗАКРУЧЕНОЇ ТЕЧІЇ У ПРЯМОЛІНІЙНІЙ КРУГЛІЙ ТРУБІ

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Abstract. Swirling flows are frequently encountered in technical devices, particularly in hydraulic and pneumatic systems. Optimal performance of these devices cannot be achieved without a thorough understanding of the physics governing these flows—physics that, as it turns out, is characterized by optimality. It is precisely these optimality properties that prove useful in situations where existing models fail to accurately describe the flow on their own. This paper provides an overview of contemporary research and achievements regarding swirling flows and highlights the challenge of describing them adequately. Based on this, the study's objective is formulated: to consolidate and systematize existing knowledge while deriving new insights–specifically, the optimality properties of swirling flows. The results demonstrate that swirling flows in a rectilinear circular pipe possess optimality properties; for instance, a flow with extremal helicity exhibits zero helicity at every point. Swirling flow where the velocity field depends solely on the radial coordinate is also optimal. For laminar flow, this entails the property of maximum flow rate, whereas for turbulent flow, the condition of maximum vorticity is also satisfied. The obtained results are compared with existing theoretical and experimental data.

Keywords: swirling flow, rectilinear pipe, optimal properties.

Introduction.

Swirling, or vortex, flows are widely encountered in nature and engineering. If the gas flow velocity does not exceed a Mach number of 0.2–0.3, such flows can be quantitatively treated as incompressible fluid flows. Modern flow classification [1] categorizes them into natural flows—formed without human intervention—and those generated through human intervention or, more broadly, by an external force. Understanding the nature (physics) of these flows is crucial for improving (optimizing) existing equipment and developing new technology that involves the generation of swirling flows.

To understand why a flow acquires a swirl in natural conditions, one must answer a single question: why? The answer is simple: such a flow must possess optimal properties—at least one. For instance, a swirling flow in a pipe containing a spiral insert exhibits the property of maximizing fluid rate [2]. However, this phenomenon (or property) does not manifest across the entire range of parameters [2]. It is associated, firstly, with a turbulent flow regime and, secondly, with the presence of a second phase (a small quantity of air bubbles). To entirely understand the physics of the phenomenon, it is necessary to recall the existence of the so-called inverse cascade of turbulent energy [3, 4]. For a long time, it was unclear why vortex flows in nature could persist for weeks or months. It was not clear where they derived the "fuel" to sustain themselves, given that turbulent energy dissipation occurs continuously within a vortex flow.

In the case of swirling flow—where the turbulent stress tensor loses its mirror symmetry—an inverse cascade of turbulent energy may occur; in simpler terms, this corresponds to flow domains characterized by negative turbulent viscosity. An inverse energy cascade is also possible in three-dimensional isotropic turbulence [5], freely decaying two-dimensional turbulence [6], and experimentally generated swirling turbulence [7]. Consequently, there arises the problem of describing swirling flow in a rectilinear circular pipe in terms of its optimality properties.

Literature Review.

Since the inverse cascade of turbulent energy allows natural vortices to maintain a compact shape, swirling flows in pipes have been modeled based on the concept of compact vortices [11, 2, 9]. These studies essentially examine two types of swirling flows: helical flows [5] and flows with helical symmetry [2, 9]. In [8], the flow is termed "predominantly" Beltrami-Gromeka, as it is not helical throughout the entire domain: near a solid wall, viscous effects preclude the existence of such a flow. In this context, reference should be made to [10], which specifically analyzes the asymptotic behavior of swirling flow near the wall of a straight, circular-section pipe.

In this paper, the optimality of swirling flow in a rectilinear pipe of constant circular cross-section is examined using a unified approach based on the calculus of variations. Furthermore, the concept of swirling flow helicity—introduced by Moffatt [11] as a measure of extremality—is employed. Helicity is a physical quantity measurable in vortex tubes [12] and possesses the property of conservation [13]. Additionally, another property of helicity is demonstrated: swirling helical flows actually observed in nature and engineering exhibit an extremal helicity value. This value is zero. In light of this, an overview of recent research on the subject is presented.

A simplified model of helical vortex dynamics in the wake of an asymmetric rotor is examined in [14]. Models involving single and dual helical vortices are discussed in [15] and [16], respectively. Other studies focus on the interaction of helical vortex flow with other flows; for instance, the interaction between a helical vortex ring and a vortex line is analyzed in [17]. Issues regarding verification—specifically concerning the existence of such flows—are addressed in [18], which investigates the development of flow in the form of helical vortex pairs through both numerical and experimental methods. Studies concerning the breakdown of helical flow in helical tubes and helical rings are presented in [19]. Information on how axial flow affects the stability of a helical vortex tube can be found in [20]. Work [21] is devoted to the same topic of stability but focuses on the experimental realization (modeling and formation) of a helical vortex.

Helical vortex flows occur during the operation of a wind turbine. Theoretical research on this topic is presented in [22]. Work [23] is dedicated to the dynamics of helical vortices and vortex rings. Flow in the form of helical vortex pairs is examined in [24]. Issues regarding the stability and structure of the Zhukovsky model for the rotor wake are addressed in [25]. The topology of helical flows is studied in [26], while the internal structure of vortex rings and helical vortices is investigated in [27]. Helical vortices can exist in a state close to equilibrium while simultaneously evolving [28]. A stability analysis within the linear approximation is provided in [29].

Research Methodology.

The aim of the study is to investigate the extremal properties of a steady swirling flow in a straight circular pipe.

This study is based on analytical methods, specifically the calculus of variations and the mathematical theory of fields.

This paper examines a class of incompressible fluid flows in which the velocity field satisfies the following relations:

$$V_r \equiv 0, \quad V_\theta = V_\theta(r), \quad V_z = V_z(r). \tag{1}$$

Essentially, two problems are considered. The first is the problem of a swirling flow with extremal helicity, and the second is a swirling flow with maximum vorticity and fluid flow rate.

Results.

Turbulent swirling flow with zero helicity

The first task is to identify, within class (1), the flow with extreme helicity:

$$J = \int_0^{2\pi}\int_0^{R}\left(rot\vec{V}\cdot\vec{V}\right)\cdot r dr d\varphi \to extr. \tag{2}$$

Turbulent stresses (or Reynolds stresses) for the flow described by (1). To understand the problem, let us present the Reynolds equations. Based on the general Reynolds equations, the form for (1) is as follows:

$$0 = \frac{d}{dr}\left(\tau_{r\theta}\right) + \frac{2}{r}\tau_{r\theta}, \tag{3}$$

$$0 = -\frac{dp}{dz} + \frac{1}{r}\frac{d}{dr}\left(\tau_{rz}\right). \tag{4}$$

Ordinary derivatives are used in equations (3)–(4). This is explained by relations (1) and the fact that the pressure is a function of the longitudinal coordinate only:

$$\frac{dp}{dz} = \Delta p = Const. \tag{5}$$

A distinctive feature of the Reynolds equations (3)–(4) is that they are expressed in terms of stresses, rather than in the conventional form involving velocity fluctuations. As will become evident, this allows for analytical solutions for the stresses. To this end, we employ the following form of equation (3):

$$\frac{d}{dr}\left(\tau_{rz}\right) = -\frac{2}{r}\tau_{r\theta}. \tag{6}$$

The general solution of equation (6) has the following form:

$$\tau_{r\theta} = \frac{Const}{r^2}. \tag{7}$$

Similarly, we use the following form of equation (4)

$$\frac{d}{dr}\left(r\tau_{rz}\right) = r\Delta p. \tag{8}$$

The general solution of equation (8) is

$$\tau_{rz} = \frac{1}{2}\Delta p r + \frac{Const}{r}. \tag{9}$$

As a result, analytical solutions (7) and (9) describing the Reynolds stresses for flow (1) have been obtained. Although these solutions exhibit a singularity at $r = 0$, they can still be used, since the flow is laminar near the axis ($r = 0$) and equations describing laminar flow should be employed there instead of equations (3)–(4).

Determination of the averaged velocity field based on the helicity extremum condition.

As it turns out, the condition of helicity extremum can determine the velocity field. Indeed, in accordance with condition (2), we find the components of the velocity curl:

$$\left(rot\vec{V}\right)_r \equiv 0, \quad \left(rot\vec{V}\right)_\theta = -\frac{dV_z}{dr}, \quad \left(rot\vec{V}\right)_z = \frac{1}{r}\frac{d\left(rV_\theta\right)}{dr}. \tag{10}$$

Substituting (10) into (2) yields:

$$J = \int_0^{2\pi}\int_0^{R}\left(V_z\frac{d\left(rV_\theta\right)}{dr} - rV_\theta\frac{dV_z}{dr}\right)drd\varphi \to extr\ . \tag{11}$$

Based on the form of the integrand in (11), we have:

$$\frac{\partial F}{\partial V_z} = \frac{d\left(rV_\theta\right)}{dr}, \quad \frac{\partial F}{\partial\left(\frac{dV_z}{dr}\right)} = -rV_\theta; \tag{12}$$

$$\frac{\partial F}{\partial V_\theta} = V_z, \quad \frac{\partial F}{\partial\left(\frac{dV_\theta}{dr}\right)} = rV_z. \tag{13}$$

The necessary condition for the extremum of functional (11) for the longitudinal component of the velocity is as follows

$$\frac{\partial F}{\partial V_z} - \frac{d}{dr}\left(\frac{\partial F}{\partial\left(\frac{dV_z}{dr}\right)}\right) = 0.$$

From this condition, the specific form follows.

$$V_z - \frac{d\left(rV_z\right)}{dr} = 0 \quad \Rightarrow \quad -r\frac{dV_z}{dr} = 0. \tag{14}$$

Similarly, for the rotational component of velocity

$$\frac{\partial F}{\partial V_\theta} - \frac{d}{dr}\left(\frac{\partial F}{\partial\left(\frac{dV_\theta}{dr}\right)}\right) = 0.$$

And therefore, the corresponding extremum condition is as follows:

$$\frac{d\left(rV_\theta\right)}{dr} - \frac{d\left(-rV_\theta\right)}{dr} = 0. \tag{15}$$

Equation (14) is equivalent to

$$-r\frac{dV_z}{dr}=0. \tag{16}$$

Its solution everywhere except (r=0) has the form:

$$V_z = Const. \tag{17}$$

From equation (15), we obtain:

$$V_\theta = \frac{Const}{r}. \tag{18}$$

Therefore, (17)-(18) is the desired solution. Since, according to (10),

$$\left(rot\vec{V}\right)_\theta = -\frac{dConst}{dr}\equiv 0,\ \left(rot\vec{V}\right)_z = \frac{1}{r}\frac{d\left(r\dfrac{Const}{r}\right)}{dr}\equiv 0$$

then (17)–(18) correspond to the minimum (zero) helicity of a swirling turbulent flow of class (1).

Steady axisymmetric swirling flow

Within the class of flows (1), let us identify a flow in which two extrema—a maximum of vorticity and a maximum of fluid discharge—are attained simultaneously.

$$J=\int_0^{2\pi}\int_0^R\left(rot\vec{V}\right)^2\cdot rdrd\varphi+\lambda\int_0^{2\pi}\int_0^R V_z\cdot rdrd\varphi\to extr. \tag{19}$$

For the flow described by (1), the components of the velocity curl (rotor) are given by expressions (10):

$$\left(rotV\right)_z=\frac{1}{r}\frac{\partial\left(rV_\theta\right)}{\partial r},\ \left(rotV\right)_\theta=-\frac{\partial V_z}{\partial r}.$$

Therefore, condition (19) takes the following form:

$$J=\int_0^{2\pi}\int_0^R\left[\left(\frac{\partial V_z}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\left(rV_\theta\right)}{\partial r}\right)^2\right]\cdot rdrd\varphi+\lambda\int_0^{2\pi}\int_0^R V_z\cdot rdrd\varphi\to extr. \tag{20}$$

Since the integrals in (20) have the same limits of integration, the overall integrand should be considered to be

$$F=\left[\left(\frac{\partial V_z}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\left(rV_\theta\right)}{\partial r}\right)^2\right]\cdot r+\lambda V_z\cdot r.$$

Analogously to formulas (12)–(13), we find the partial derivatives:

$$\frac{\partial F}{\partial V_\theta}=\frac{2V_\theta}{r}+2\frac{\partial V_\theta}{\partial r},\quad \frac{\partial F}{\partial\left(\dfrac{dV_\theta}{dr}\right)}=2V_\theta+2r\frac{dV_\theta}{dr}. \tag{21}$$

Substituting the expressions just obtained into the necessary condition for the extremum of the functional (20). Substituting (21) into (13) leads to the following equation:

$$\frac{d^2V_\theta}{dr^2}+\frac{1}{r}\frac{dV_\theta}{dr}-\frac{V_\theta}{r^2}=0. \tag{22}$$

Equation (22) is identical to the Navier-Stokes equation for the problem of fluid rotation when only the rotational velocity component is present and the velocity field depends solely on the radial coordinate. Recall that the general solution to (22) has the following form:

$$V_\theta = C_1 r + C_2 \frac{1}{r}.$$

It should be noted that this solution has components corresponding to both laminar and turbulent flow [30]. Let us use the second necessary condition for the extremum of functional (14).

Since

$$\frac{\partial F}{\partial V_z} = r\lambda(r); \qquad \frac{\partial F}{\partial\left(\frac{dV_z}{dr}\right)} = 2r\frac{dV_z}{dr},$$

then equation (14) takes the following explicit form:

$$r\lambda(r) - 2\frac{dV_z}{dr} - 2r\frac{d^2V_z}{dr^2} = 0. \tag{23}$$

The solution to equation (23) in the case of

$$\lambda(r) = 0$$

has the following form, well known for turbulent flow:

$$V_z = C_1 \ln r + C_2.$$

So, if we do not require the maximum flow rate condition from the flow, but only the maximum turbulence condition, then the flow is described by the well-known logarithmic law (this fact was first obtained earlier for ordinary flow in a pipe – without rotation; the corresponding article was submitted to the Journal of Turbulence). But in the case of satisfying the second extremity condition (maximum flow rate), the velocity field will be different. So, when

$$\lambda(r) = C_{lam} / r$$

the solution to equation (23) contains a solution well known in hydraulics:

$$V_z = C_{lam}\frac{r^2}{4} + C_1 \ln r + C_2. \tag{24}$$

For laminar flow in a circular pipe

$$C_1 = 0.$$

Note that for flow between concentric pipes, all three terms in (24) are non-zero. To understand the physics of the resulting solution, it should be treated as a piecewise continuous function: in the laminar flow region, the constant multiplying the logarithm is zero, whereas in the turbulent flow region, the logarithmic component must be included. Furthermore, other functions—such as Lagrange multipliers—may be specified if necessary to match an experimental profile.

Conclusions.

Extremal properties of swirling flow—specifically helicity, vorticity, and flow rate—were investigated. The condition for extremal helicity allows for the derivation of the turbulent flow core (excluding the boundary layer) without recourse to the Boussinesq

hypothesis. Based on the derived analytical expression for the velocity field (which is also utilized in centrifugal pump theory), Reynolds stresses can be readily determined from the Reynolds equations. However, the principal conclusion drawn from this part of the study is that an optimal helical flow can possess only zero helicity—and this applies throughout the entire flow domain.

According to the generalized criterion of inertial stability [31], the velocity field in a turbulent flow must be such that the azimuthal velocity decreases more rapidly than the hyperbolic law dictates. Comparing equation (18) with this premise reveals that a real turbulent flow in a pipeline possesses an additional characteristic causing its velocity field to decay more sharply than a hyperbola. Such a vortex flow model is already known: the quasi-point vortex.

In addition to zero helicity, a swirling flow in a rectilinear circular pipe possesses two other optimal properties: maximum vorticity (or, more precisely, its square) and maximum fluid flow rate. It is the simultaneous application of these two conditions that yields well-known solutions corresponding to both laminar and turbulent incompressible fluid flows. In this context, the general solutions for azimuthal velocity satisfy the Navier-Stokes equations, while the longitudinal (axial) velocity satisfies an equation whose solution encompasses both laminar and turbulent flow By setting a particular integration constant to zero, it can be used separately for laminar and turbulent flows. Thus, both laminar and turbulent swirling flows in a rectilinear circular pipe are optimal.

Future research should focus on a more thorough investigation of the boundary layer in swirling flow and the identification of potential flow regions characterized by negative turbulent viscosity, which would further substantiate the optimal nature of swirling flow.

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ОПТИМАЛЬНІ ВЛАСТИВОСТІ НЕСТИСЛИВОЇ ЗАКРУЧЕНОЇ ТЕЧІЇ У ПРЯМОЛІНІЙНІЙ КРУГЛІЙ ТРУБІ

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Анотація

Закручені течії часто зустрічаються у технічних пристроях, зокрема у гідравлічних та пневматичних. Досконала робота цих пристроїв не може бути досягнута без повного розуміння фізики цих течій. А вона, ця фізика, як виявляється, має оптимальний характер. Дійсно, адже ніхто, крім законів природи, не примушує воду текти у гірських струмках у вигляді закрученої течії (спостереження австрійського лісничого Віктора Шаубергера), а воду у ванні набувати закрутку. Раніше були проведені експериментальні дослідження у лабораторії. І ці дослідження дійсно виявили існування таких режимів турбулентної течії рідини у трубі із примусовою закруткою. Крім того, стало очевидним, що природа таких течій є дуже складною. Тому саме властивості оптимальності допомагають там, де існуючі моделі неспроможні самостійно впоратися із коректним описам течії. В роботі надано огляд сучасних робіт і досягнень щодо закручених течій, висвітлена проблема їх адекватного опису. На підставі цього сформульована мета даної роботи. Вона полягає у об'єднанні та систематизації вже відомого знання та отримання нового ÷ властивостей оптимальності закрученої течії. До цього вже були отримані такі результати, як відмінність ламінарної течії у примежовому шарі у випадках внутрішньої та зовнішньої задач. Також виявлено різницю між природою турбулентних течій у примежових шарах у круглій трубі та на плоский пластині. Звичайна розвинена турбулентна течія у круглій трубі має логарифмічний профіль, так само як і течія на плоскій пластині. Але якщо у першому випадку виконується умова максимального завихрення (точніше квадрату завихрення), то у другому випадку логарифмічний профіль є результатом виконання двох екстремумів ÷ максимуму квадрату виразу, що описує інерційну нестійкість, та мінімуму витрати рідини. Зазначені щойно ідеї та отримані за ними результати слугували основою для наведених у даній роботі досліджень. В результаті показано, що закручені течії у прямолінійній трубі кругового перерізу мають властивості оптимальності. Так течія із екстремальною спіральністю має рівну нулеві спіральність у кожній точці. Що стосується закрученої течії, у якої поле швидкості залежить лише від радіальної координати, то вона теж оптимальна. Для ламінарної течії – це властивість максимальної витрати, а для турбулентної течії додається ще максимальне значення квадрату завихрення. Отримані результати порівняються з вже відомими теоретичними та експериментальними даними.

Ключові слова: закручена течія, прямолінійна труба, оптимальні властивості.