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THEORETICAL BASIS OF ELECTRIC DRIVES WITH INHERENT NONLINEARITIES

Summary. In articles, the theoretical justification of electric drives is presented, whose essential influence on dynamic and static characteristics stems from inherent nonlinearities of electromagnetic, mechanical, and control nature. It has been shown that a real electric drive is not expedient to be considered as a set of linear links from small deviations parameters, but as a complex nonlinear electromechanical system, whose is forming magnetic saturated, dry and viscous friction, backlash and gaps in gears, limitations on voltage and current of the power converter, nonlinear load, hysteresis, dead zones and discreteness management influences. It is justified that linearised models are valid only in the vicinity of the operating point and do not provide adequate performance across a wide range of operating modes, including start, reverse, low-speed movement, torque boost, and operation under variable disturbances. Proposed generalised approach to mathematics description of electric drive in space states, taking into account basic nonlinear dependencies. The feasibility of integrated use is shown. local linearization, energy methods, Lyapunov functions, phase analysis, and modern nonlinear algorithm management. Received results from the theoretical basis for the synthesis of high-precision, robust and energy-efficient electric drive systems.

Keywords: electric drive, nonlinearity, electric machine, mathematical model, magnetic saturation, friction, backlash, robust control, stability, power converter.

Problem statement. Electric drive, in modern technical understanding, covers the electric machine, the power plant converter, the mechanical transmission, the load, and the control system. It is a multicomponent structure that makes him the only electromechanical system in which transformation energy, formation torque, motion transmission, and coordinate adjustment are interconnected [1].

However, in real conditions, the operation of such a system is almost never linear. Magnetic characteristics of the core change due to saturation; mechanical transmission contains clearances; friction has Coulomb, viscous, and mixed components; and regulators and converters operate under restricted voltage, current, and frequency switching [2]. Under these conditions of application, exclusively linear models lead to a simplification that is permissible only for a narrow range around the operating point. For a wide range of modes, a real electric drive needs a nonlinear theoretical description [10].

Analyse the last research and publications. In modern research, nonlinearities in electric drive systems are considered one of the main factors that restrict precision adjustment, smoothness, and stability. For mechanical parts, particular importance is given to backlash nonlinearity, dry and mixed friction, and elastic deformations in gears [3].

For electromagnetic parts, the primary source of nonlinearity is magnetic saturation [4], which causes variations in inductance and distortion of transitional processes [5]. Separate directions constitute research on nonlinear friction, because it directly affects not only the accuracy of management but also the identification of procedures and parameters in systems [6]. Development of modern PMSM – and asynchronous drives accompanied active implementation methods *FEEDBACK LINEARIZATION, BACKSTEPPING, SLIDING MODE, ADAPTIVE SCHEMES* [7–9].



Formulation of the purpose of the article (statement of the task). The purpose of the article is to provide a theoretical justification for electric drives with inherent nonlinearities by developing a generalised nonlinear description of electromechanical systems, identifying the main sources of nonlinearities, evaluating their impact on the drive's dynamic properties, and defining appropriate approaches to the analysis and synthesis of control systems.

The main part. In general case electric drive expedient describe in space states as a nonlinear system of the form:

$$x = f(x, u, p, t), \quad y = g(x, u, p, t) \quad (1)$$

where x is the state vector, which may include currents, flux linkage, angular velocity, angle of rotation, and deformations of mechanical links;

u is the control influence;

p is the object parameters. This form is natural for describing electric drives when it is necessary to simultaneously account for electromagnetic, mechanical, and control constraints [1].

For a generalized electric drive, the electrical and mechanical subsystems can be represented by the equations:

$$u = R(T, i)i + \frac{d\psi(i, \theta)}{dt} + e(\omega, \psi), \quad (2)$$

$$J(\theta) \frac{d\omega}{dt} = M_e(i, \psi) - M_c(\omega, \theta) - M_f(\omega) - M_d(t) \quad (3)$$

where $R(T, i)$ – resistance, depending on temperature and mode;

$\psi(i, \theta)$ – flux linkage as a nonlinear function of current and position;

M_e – electromagnetic moment;

M_c – load moment;

M_f – friction moment;

M_d – disturbing moment. Already from this form, it is clear that Nonlinearity in an electric drive is not a secondary effect but an inherent property of physics.

The first fundamental source of nonlinearity is magnetic saturation [4, 5]. In linear approximations, the flux linkage is often given as $\psi = Li$; however, the real magnetic system of the machine has a nonlinear magnetisation curve, and the mutual inductance changes with saturation. For induction machines and polyphase drives, consideration of saturation and crossover saturation substantially affects transient currents and voltages, as well as the accuracy of models.

The second principle factor is nonlinearity mechanical channel. Dry friction can be described approximated as [6]:

$$M_f(\omega) = M_{c0} \operatorname{sgn}(\omega) + b\omega \quad (4)$$

where M_{c0} is the Coulomb component;

$b\omega$ – binding component. Such a model is already unevenly smooth and, in the low-speed zone, generates effects such as sticking, microvibrations, and reduced positioning accuracy.

Third, the source is backlash, gear and elastic transmission gaps [3]. They generate dead zones, delay torque transmission, shock absorber phenomena during changes in direction of movement, and additional oscillatory modes. Since BACKLASH is an ambiguous nonlinearity with memory, it cannot be reduced to a simple static correction and must be considered separately in the model.

Fourth group nonlinearities related to the force converter and control system [7–9]. Current and voltage limitations, saturation regulators, PWM discreteness, digital computing delays, and *anti-windup effects* actually change the system's structure *depending* on the regime.



Fifth nonlinearity is exactly load. For many mechanisms, the moment of resistance is not constant and depends from speed in the form of:

$$M_c(\omega) = M_0 + k\omega^n \quad (5)$$

For pumps and fans, the typical law is the quadratic law; for transport and lifting mechanisms, piecewise linear or combined characteristics.

From here follows key theoretical position: linearization has local character. If for a stationary point (x_0, u_0) the following holds: condition $f(x_0, u_0) = 0$, then in the neighborhood this point the system can be presented in the first approximate as :

$$\Delta x = A\Delta\dot{x} + B\Delta u, \quad (6)$$

where

$$A = \left. \frac{df}{dx} \right|_{x_0, y_0}, \quad B = \left. \frac{df}{du} \right|_{x_0, y_0}. \quad (7)$$

This simplification is useful for controller synthesis, but it loses its adequacy when passing through zero speed, entering saturation, changing load, or structurally changing the converter mode. That is why, for a wide operating range of an electric drive, a nonlinear model is not an alternative, but a basic form of description.

Question: It is advisable to consider the stability of such systems on the basis of the energy approach and Lyapunov functions [10]. For an electric drive, a natural candidate for the Lyapunov function is the sum of the electromagnetic and mechanical energies. If the derivative of this function along the system's trajectories is negative, we can draw conclusions about sustainability or asymptotic regime stability.

From a control synthesis perspective, it means that traditional linear PI and PID controllers are considered a special case suitable for modes with relatively weak nonlinearities. If available, essential changes parameters, load, saturation, and disturbances become more justified, robust and nonlinear methods: SLIDING MODE, BACKSTEPPING, FEEDBACK LINEARIZATION, DISTURBANCE OBSERVER – BASED CONTROL, ADAPTIVE SCHEMES.

So, theoretically, the justification of electric drives with inherent nonlinearities consists in moving from simplified linear interpretations to a systemic nonlinear description. Such an approach should combine a physically correct model, local linearization for engineering synthesis, energy analysis, stability and numerical modelling characteristic modes.

Conclusions. It was found that nonlinear electric drives have a fundamental nature and arise in all main subsystems: electromagnetic, mechanical, power and control [4–7].

It has been proven that magnetic saturation, nonlinear friction, backlash, gaps, voltage and current constraints, and the load's nonlinear nature significantly alter the drive's dynamic characteristics and cannot be adequately captured by a single global linear model.

It is justified that linearised models are acceptable only for local analysis near the operating point, whereas over a wide range of modes, the electric drive must be described by a nonlinear state-space model.

Theoretical analysis indicates that a comprehensive approach combining physically meaningful nonlinear modelling, local linearization, Lyapunov methods, phase analysis, and robust control synthesis is advisable.

The received position can serve as a basis for further research and the design of high-precision, adaptive, and energy-efficient electric drive systems.

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ТЕОРЕТИЧНЕ ОБҐРУНТУВАННЯ ЕЛЕКТРОПРИВОДІВ З ПРИТАМАННИМИ НЕЛІНІЙНОСТЯМИ

Анотація

У статті здійснено теоретичне обґрунтування електроприводів, у яких суттєвий вплив на динамічні та статичні характеристики мають притаманні нелінійності електромагнітної, механічної та керувальної природи. Показано, що реальний електропривод доцільно розглядати не як сукупність лінійних ланок із малими відхиленнями параметрів, а як складну нелінійну електромеханічну систему, поведінка якої формується магнітним насиченням, сухим і в'язким тертям, люфтами та зазорами у передачах, обмеженнями напруги і струму силового перетворювача, нелінійністю навантаження, гістерезисом, мертвими зонами та дискретністю керувальних впливів. Обґрунтовано, що лінеаризовані моделі є коректними лише в околі робочої точки і не забезпечують належної адекватності під час аналізу широкого діапазону режимів, зокрема пуску, реверсу, малозшвидкісного руху, форсування моменту та роботи в умовах змінних збурень. Запропоновано узагальнений підхід до математичного опису електропривода у просторі станів з урахуван-



ням базових нелінійних залежностей. Показано доцільність комплексного використання локальної лінеаризації, енергетичних методів, функцій Ляпунова, фазового аналізу та сучасних нелінійних алгоритмів керування. Отримані результати формують теоретичну основу для синтезу високоточних, робастних та енергоефективних електроприводних систем.

Ключові слова: електропривод, нелінійність, електрична машина, математична модель, магнітне насичення, тертя, люфт, робастне керування, стійкість, силовий перетворювач.