Assessment of the necessity of cooling of the Stirling engine expansion piston

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Abstract

Heat balance of the Stirling engine expansion piston is investigated. This balance considers heat flows that are applied to the expansion piston from outside parts, generated by friction in the piston-liner contact zone, and rejected to the cooling medium. The average heat transfer coefficient is obtained by means of division of a predicted value of the thermal conductance by cooled area of the expansion piston. Dimensionless variables are derived from an expression for determination of the thermal conductance, and the dimensionless chart is presented.

Nomenclature

A = area (m^2)
h = heat transfer coefficient ($Wm^{-2}K^{-1}$)
hA = thermal conductance on the side of gas or liquid (W/K)
P = heat flux (W)
R = thermal resistance (K/W)
T = temperature (K)
X_P, $X_{\Delta T}$, Y = dimensionless variables as defined in section 4
z = auxiliary function in eq. (6) (W/K)

Indices
cl = cylinder liner
cm = cooling medium
dd = displacer dome
ep = expansion piston
fr = friction
rj = rejected
wg = working gas
wg,e = working gas (for expansion space)

1. Introduction

Expansion and compression pistons of the Stirling engine are loaded both mechanically and thermally, but with different interrelation of these kinds of loading. It is seen from general considerations regarding Stirling engines [1].

Mechanical loads applied to expansion and compression pistons are similar and caused by the following matters: (1) pressure differences between opposite surfaces; (2) inertia forces generated by the reciprocating motion of pistons; (3) friction forces at contact zones of pistons and cylinder walls; etc.

Thermal load applied to the expansion piston far exceeds that applied to the compression one. It caused with much higher temperature of the working gas near the expansion piston than that near the compression one. Therefore, conditions of operation are more difficult for the expansion piston. With limited heat rejection from it, its temperature can exceed reasonable limits, causing its premature failure. This is similar to such problem existing for pistons of internal combustion engines (ICEs) [2]. However, due to gas exchange the ICE piston do not contact with constantly hot working medium, as does the Stirling engine expansion piston. (It is assumed that the working gas in the expansion space has elevated temperature, being not below 300…400^{o}C.) Available technical literature rarely considered thermal loading of the expansion piston.

The purpose of the present paper is to develop an analytical method allowing to assess whether the Stirling engine expansion piston needs specially organized external cooling.

2. Heat balance and thermal conductance of the expansion piston

It is assumed that the expansion piston: (1) is connected to the 'expansion' connecting rod through the piston pin (like in ICEs); (2) is connected to the displacer dome which constantly contacts with hot working gas; (3) slides along the expansion cylinder liner; (4) is properly sealed, for instance, by a set of 'expansion' piston rings. Also it is assumed that temperature differences inside selected areas of the working gas, the cooling medium and the expansion piston are small with comparison to corresponding characteristic temperatures.

For the expansion piston, equation of the heat balance can be written in terms of heat fluxes (W) [3]:

$$P_{dd} + P_{cl} + P_{fr} = P_{rj} \tag{1}$$

where terms in the left-hand part relate to heat inputs applied to the expansion piston, and the right-hand term denotes the heat flux rejected to the cooling medium.

Input heat fluxes (P_{dd} and P_{cl}) are determined by division of characteristic temperature differences (or 'excessive temperatures') by corresponding full thermal resistances as follows:

$$P_{dd} = (T_{wg,e} - T_{ep})/(R_{dd} + 1/(hA)_{wg}); \quad P_{cl} = (T_{cl} - T_{cm})/(R_{cl} + 1/(hA)_{rj}) \tag{2; 3}$$

The input heat flux generated by friction in the piston-liner contact zone, P_{fr}, is proportional to average product of the transversal force and the piston velocity. For suitability of further algebra, sum of heat inputs, $P_{dd} + P_{fr}$, will be denoted as P_{dd+fr}. It does not depend on the expansion piston thermal conductance, $(hA)_{rj}$, which affects the cylinder liner heat input, P_{cl}, and the rejected heat flux, P_{rj}.

The heat flux P_{rj} is determined as product of the expansion piston thermal conductance, $(hA)_{rj}$, and the 'effective' excessive temperature, $(T_{ep} - T_{cm})$, of this piston. Substitution of expressions for P_{cl} and P_{rj} into eq. (1) gives

$$P_{dd+fr} + (T_{cl} - T_{cm})/(R_{cl} + 1/(hA)_{rj}) = (hA)_{rj}(T_{ep} - T_{cm}). \tag{4}$$

After rewriting this in form of a quadratic equation, the thermal conductance, $(hA)_{rj}$, can be derived as follows:

$$R_{cl}(T_{ep} - T_{cm})(hA)_{rj}^{2} - (R_{cl}P_{dd+fr} + (T_{cl} - T_{ep}))(hA)_{rj} - P_{dd+fr} = 0; \tag{5}$$

$$(hA)_{rj} = z\left(1 + \sqrt{1 + \frac{2}{z\left(R_{cl} + \dfrac{T_{cl} - T_{ep}}{P_{dd+fr}}\right)}}\right) \tag{6}$$

where

$$z = \frac{P_{dd+fr}R_{cl} + (T_{cl} - T_{ep})}{2R_{cl}(T_{ep} - T_{cm})} = \frac{P_{dd+fr} + \dfrac{T_{cl} - T_{ep}}{R_{cl}}}{2(T_{ep} - T_{cm})}. \tag{7}$$

For rough estimation of $(hA)_{rj}$, it can be assumed that the sum of two heat inputs, P_{dd+fr}, is negligible with comparison to the cylinder liner heat input, P_{cl}. With this assumption, the expansion piston thermal conductance is given:

$$(hA)_{rj} \approx \frac{T_{cl} - T_{ep}}{2R_{cl}(T_{ep} - T_{cm})}\left(1 + \sqrt{1 + \frac{4P_{dd+fr}R_{cl}(T_{ep} - T_{cm})}{(T_{cl} - T_{ep})^{2}}}\right). \tag{8}$$

For very idealized case of zero P_{dd+fr}, $(hA)_{rj}$ is greater than the first term of eq. (8) by two times.

If cooled area of the expansion piston and its thermal conductance, $(hA)_{rj}$, are known, the heat transfer coefficient, h_{rj}, can be determined by simple arithmetics. This will give a 'predicted' value of the coefficient, which should be compared with expected values for available methods of the cooling.

3. Numerical examples

A numerical example allows to obtain predicted values of $(hA)_{rj}$ from equations (8) and (6). Assumed initial data: temperatures T_{cl} = 500, T_{ep} = 400, T_{cm} = 300 K; cylinder liner heat resistance R_{cl} = 0,01 K/W; sum of displacer dome and friction heat inputs P_{dd+fr} = 1000 W. With these values to be substituted, eq. (8) returns a sample approximate value of the thermal conductance:

$$(hA)_{rj} \approx \frac{100}{2\cdot 0,01\cdot 100}\left(1+\sqrt{1+\frac{4\cdot 1000\cdot 0,01\cdot 100}{100^2}}\right)\approx 50\cdot 2,2 = 110\ W/K.$$

At the same initial data, eq. (6) returns a more accurate sample value:

$$z=\frac{1000\cdot 0,01+100}{2\cdot 0,01\cdot 100}=55\ W/K;\quad (hA)_{rj}=55\left(1+\sqrt{1+\frac{2}{55\left(0,01+\frac{100}{1000}\right)}}\right)\approx 55\cdot 2,15\approx 120\ W/K.$$

It is seen that the difference between both sample values is much less than the more accurate value of $(hA)_{rj}$. Therefore, eq. (8) can be used instead eq. (6), at least, in certain cases.

There is a question about accessibility of obtained sample values of $(hA)_{rj}$. Are they normal? For assumed expansion piston excessive temperature of 100 K, the rejected heat flux, P_{rj}, amounts 11…12 kW – possibly well beyond useful power of the engine. For diameter of the piston near 0,1 m, its minimum cooled area can be near 0,01 m^2. This requires a value of the heat transfer coefficient, exceeding 10^4 Wm^{-2}K^{-1}!

So, the obtained sample values of $(hA)_{rj}$ are not normal. To get more appropriate ones, at first it is necessary to decrease the expansion piston excessive temperature. This can be achieved by thermal insulation of the expansion piston and its cylinder liner. Secondly, it seems useful to extend the piston cooled area by means of some kind of fins.

For instance, if these expedients allow having the piston excessive temperature of 20 K and the cooled area of 0,1 m^2, $(hA)_{rj}$ increases to 510 W/K (eq. (8)) and a required value of h_{rj} decreases to 5100 Wm^{-2}K^{-1}. It seems still superfluous, and the rejected heat flux, P_{rj}, is still large - near 10 kW.

Therefore, excessive temperature of the cylinder liner should also be decreased. If it is decreased from 100 to 50 K, eq. (8) returns

$$(hA)_{rj} \approx \frac{50}{2\cdot 0,01\cdot 20}\left(1+\sqrt{1+\frac{4\cdot 1000\cdot 0,01\cdot 20}{50^2}}\right)\approx 270\ W/K.$$

A required value of h_{rj} decreases to 2700 Wm^{-2}K^{-1} (at A_{rj} = 0,1 m^2) and is achievable, if liquid coolant (for instance, water) is sprayed onto extended cooled surface of the expansion piston. The rejected heat flux, P_{rj}, amounts near 5 kW, allowing using the Stirling engine as a part of the local heating system.

4. Dimensionless presentation

Several dimensionless variables can be derived from eq. (8), for instance:

$$Y=(hA)_{rj}R_{cl};\quad X_{\Delta T}=\frac{T_{cl}-T_{ep}}{T_{ep}-T_{cm}};\quad X_P=\frac{P_{dd+fr}R_{cl}}{T_{cl}-T_{ep}}.$$

Using them, equations (6) and (8) can be presented in dimensionless form:

$$Y=0,5X_{\Delta T}(1+X_P)\left(1+\sqrt{1+\frac{4}{X_{\Delta T}(1+X_P)(1+1/X_P)}}\right)\approx 0,5X_{\Delta T}\left(1+\sqrt{1+4X_P/X_{\Delta T}}\right). \quad (9)$$

What can be values of these variables? $X_{\Delta T}$ varies near unity (for instance, between 0,3 and 3). With product $P_{dd+fr}R_{cl}$ being possibly within 10…100 K and the cylinder liner excessive temperature being within 20…100 K, X_P varies also near unity. From the simplified form of eq. (9) it is seen that Y is proportional to $X_{\Delta T}$, when $X_{\Delta T}$ is large or X_P is small. So it can be expected that at lower X_P and larger $X_{\Delta T}$, Y can approach such values of $X_{\Delta T}$ (for instance, 3). When $X_{\Delta T}$ is small (for instance, 0,3), Y more explicitly depends on X_P:

$$X_P = 0,2;\quad Y=0,5\cdot 0,3(1+0,2)\left(1+\sqrt{1+\frac{4}{0,3(1+0,2)(1+1/0,2)}}\right)\approx 0,5;$$

$$X_P = 5;\quad Y=0,5\cdot 0,3(1+5)\left(1+\sqrt{1+\frac{4}{0,3(1+5)(1+1/5)}}\right)\approx 2,4.$$

It is seen from these calculations that increase of X_P by 25 times causes increase of Y by near 5 times, that is, at lower $X_{\Delta T}$, Y is proportional to $\sqrt{X_P}$.

Maximum values of Y are expected, when both $X_{\Delta T}$ and X_P are large. For instance, at $X_{\Delta T} = 3$ and $X_P = 5$ eq. (9) returns $Y \approx 19$. From that mentioned in section 3 of this paper it is seen that such combinations of dimensionless variables are not useful, as at certain value of R_{cl} higher Y means larger, hardly achievable values of the thermal conductance, $(hA)_{rj}$.

A dimensionless chart where Y is plotted versus $X_{\Delta T}$ at different values of X_P is shown in Fig. 1.

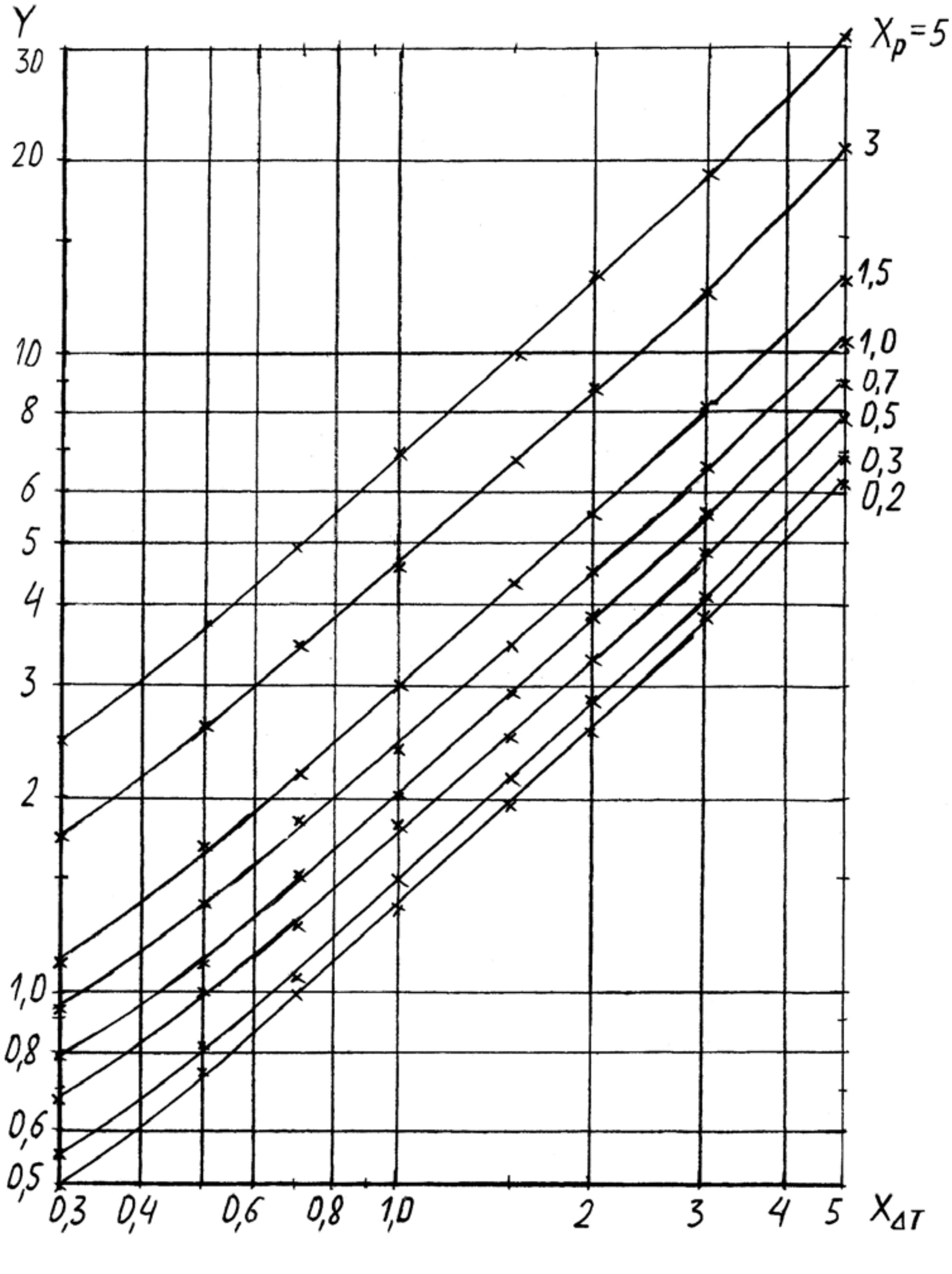

Figure 1 Dimensionless chart: Y plotted versus $X_{\Delta T}$ at different values of X_P

It is seen from the chart that, contrary to the above expectation, values of Y are significantly greater than $X_{\Delta T}$ even at $X_P = 0,2$. Central parts of plotted curves are close well to straight lines (for logarithmically marked axes), so an exponential function seems to be good approximation for these curves at the interval of $X_{\Delta T} = 0,5\ldots3$.

Least-square fitting of parameters of the exponential function

$$Y = aX_{\Delta T}^{b} \tag{10}$$

for each curve on Fig.1 gives relatively stable values of the exponent, $b \approx 0{,}87\ldots0{,}91$, with the minimum value (b_{min} = 0,867) obtained at $X_P = 1$. A value of the coefficient, a, depends on X_P also like the exponential function:

$$a = 2{,}67 X_P^{0{,}496} \, . \qquad (11)$$

Values of the correlation coefficient for these equations are high: 0,9997 for equation 10 and 0,987 for eq. (11). Beyond the interval of $X_{\Delta T}$ = 0,5…3, it is better to use dimensionless eq. (9).

The last numerical example in section 3 gives a value Y = 2,7 for such values of dimensionless variables: $X_{\Delta T}$ = 50/20 = 2,5; X_P = 1000∗0,01/50 = 0,2. The chart (based on the not simplified dimensionless eq. (9)) returns a value of Y being slightly greater than 3. This is so, because the previously made approximation $(1+X_P) \approx 1$ appears too rough. Hence a simplified form of eq. (9) is better presented by the following equation:

$$Y \approx 0{,}5 X_{\Delta T} (1 + X_P)\left(1 + \sqrt{1 + 4X_P / X_{\Delta T}}\right). \qquad (12)$$

Conclusion

For a simplified form of the heat balance of the expansion piston (eq. (1)), the thermal conductance of this piston (in direction to the cooling medium) is determined by equations (6) and (7). Equation 8 gives underestimated values of the thermal conductance and can be used only for initial assessments.

To obtain practically achievable values of the piston thermal conductance without getting too large values of the rejected heat flux, it seems useful to decrease excessive temperatures of the expansion piston and the cylinder liner, and to extend the piston cooled area by means of some kind of fins.

The dimensionless presentation of dimensional equations allows reducing the problem to three-variable one, with the most accurate dimensionless equation being the left part of eq. (9), before sign '≈'. For quicker assessments of the dimensionless thermal conductance of the expansion piston, *Y,* the dimensionless chart (Fig. 1) is useful.

References

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