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DETERMINE OF GEOMETRICAL CHARACTERISTICS RANGES OF THE DISCRETELY PRESENTED CURVE WITH LAW OF CURVATURE CHANGE

Summary. The conditions modeling of curve with a monotonous change of curvature and the scheme of the destination of the provisions tangents at which the problem of modeling the contours with a monotonous change of curvature has a solution is considered in this article.
Keywords: discretely represented curve (DRC), the tangent, normal, basic triangle, radius of curvature, monotonous curvature change.

Problem statement

The main functional characteristic of dynamic surfaces is regulated nature of flow of the medium. At designing dynamic surfaces are important lines which connect surfaces geometry with the physical side of its functional purpose. Most often it is necessary to ensure the second order of the smoothness and regular change of curvature along the contours of which are elements of the surfaces carcass. In this case it is advisable to modeling the curve locally in areas which are limited to the initial points. The resulting areas of monotonous curves connecting together with the order of smoothness are not lower than the second. In order to ensure a given nature of the change curvature along the contour it is necessary to assign position of the tangent in the initial points at which the problem of modeling the curve with a monotonous curvature change has a solution.

Analysis of the previous researches

The method of determining the possibility of modeling of the curve with monotonous change of curvature based on a given points set is developed in the work [1]. Necessary condition is the location of the initial points such that the radiuses of the circles which define three successive points, change monotonically along the row.

Methods for formation of curves segment along which the curvature changes monotonically proposed in the works [2, 5]. The conditions

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of modeling segment of curve are regular change of curvature, given positions of tangents and radiuses of curvature in the points which limit the segment. The method of determining of possible locations ranges of the thickened point on the segment of the monotonous curve is developed in the article [2]. Previously determined the position of the normal and the center of curvature which correspond to the point of thickening.

The method of determining of thickening point of the monotonous DRC on the given positions of tangents and radiuses of curvature at the initial points is proposed in the article [5]. At formation of the curves segment the point of thickening and the tangent, which passes through it, are defined within the basic triangle. The basic triangle limited to tangents that pass through two consecutive points and the chord that connects these points. The method, which made possible using parameters of the basic triangles for identify the ranges of curvature values of the monotonous curve, is developed in the work [3]. In order to using these methods to modeling a contour of the points it is necessary to assign such characteristics in the initial points (positions of normals, tangents, positions of the centers of curvature, values of the radiuses of curvature), in which the task of forming of segments curves with a monotonous change of curvature and connecting of these segments with the second order smoothness has a solution.

The formulation of purpose of the article (task statement)

The purpose of the article is to develop a method for determining of the range positions tangents and values of curvature in the initial points of the monotonous DRC and determining of location area of contours that meets the above specifications.

Basic part

Suppose that monotonous curve is modeled by the method of thickening. Let consider the segment $(i \ldots i+1)$. In the initial points are given positions of tangents t_i, t_{i+1} and normals n_i, n_{i+1} respectively (ill. 1).

Determine the conditions on the location of t_i, t_{i+1} and n_i, n_{i+1}, when it is possible to provide a monotonous increase of the radiuses of curvature at the segment $(i \ldots i+1)$.

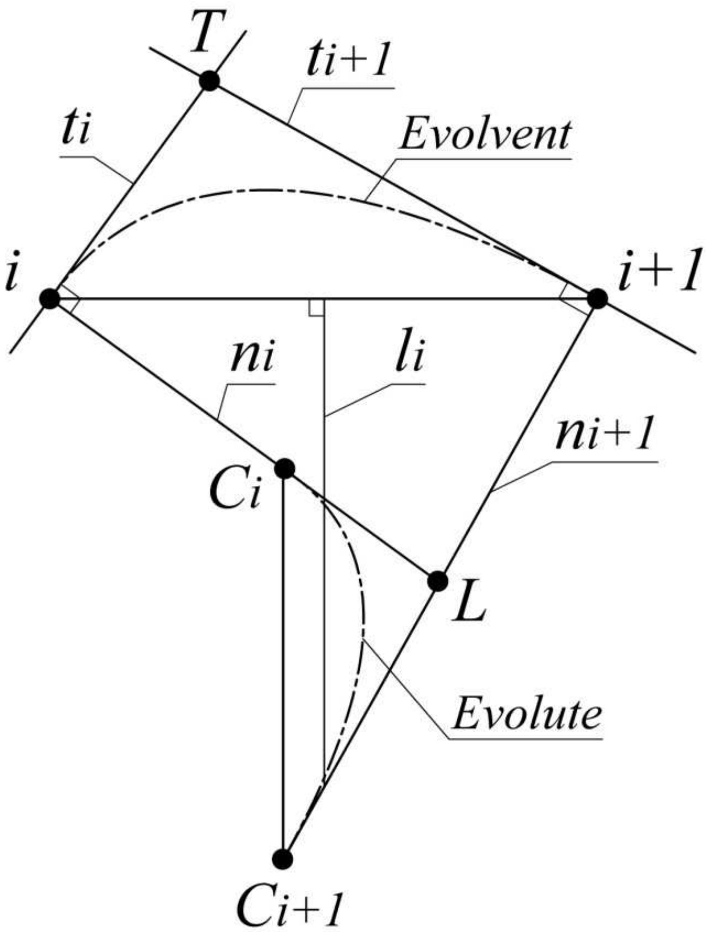

Illustration 1

Let denote the centers of curvature in the points i and $i+1$ as C_i and C_{i+1} respectively, and the point of intersection of normals as L. The triangle $C_i;L;C_{i+1}$ will be called a triangle of the centers of curvature [2].

The radiuses of curvature of the DRC in the points i and $i+1$ are respectively:

$$R_i = |C_i;i|, \qquad R_{i+1} = |C_{i+1};i+1|.$$

If along a curve on a segment $(i \ldots i+1)$ radiuses of curvature are monotonically increasing, the evolute of the curve $(C_i \ldots C_{i+1})$ is convex curve which is tangent to n_i and n_{i+1} in the points C_i and C_{i+1} respectively [4].

At the same time the length of the evolute (ΔR) is equal to the difference between the radius of the curvature at the initial points:

$$\Delta R = R_{i+1} - R_i.$$

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Thus evolute located inside the triangle $C_i;L;C_{i+1}$ and the value of its length satisfies the following conditions:

$$\Delta R > |C_i;C_{i+1}|; \quad (1)$$

$$\Delta R < |C_i;L| + |L;C_{i+1}|, \quad (2)$$

$|C_i;C_{i+1}|$, $|C_i;L|$, $|L;C_{i+1}|$ – are sides lengths of the triangle of the centers of curvature $C_i;L;C_{i+1}$.

The condition (2) can be written as

$$|C_{i+1};i+1| - |C_i;i| < |C_i;L| + |L;C_{i+1}|,$$

$$\text{or } |i;L| > |L;i+1|. \quad (3)$$

With such an arrangement of normals the tangents t_i, t_{i+1} and chord $[i;i+1]$ determines the basic triangle $i;T;i+1$ with an aspect ratio that satisfies the condition:

$$|i;T| < |T;i+1|. \quad (4)$$

Location of normals and tangents in which the following requirements (3) and (4), respectively, are essential for the formation of a curve with a monotonous increase of curvature on the segment $(i \ldots i+1)$. Conditions (3) and (4) may be used as a criterion for determining the position of the tangents (normals) in the initial points according to the conditions of the problem.

In modeling of the DRC along the points positions of the tangents in initial points must be assigned in such a way that the condition (4) holds for all segments of the curve. Lets define the range of positions of the tangent t_i.

To do this the analysis phase of the initial points every three consecutive points is held circles. Obtained circles is called as contiguous circles. Suppose that through the point i are passing circles $ПК_{i-1}$, $ПК_i$ and $ПК_{i+1}$. In order to modeling the DRC with a monotonous change of curvature, the range of possible locations of the tangent is the angle that is limited by the tangents of the respective contiguous circles. One boundary is tangent to the circle $ПК_i$ ($t_{ПК i}$) and the second - a tangent that is closer to $t_{ПК i}$: tangent to circle $ПК_{i-1}$ or to circle $ПК_{i+1}$ [1].

After determining the ranges of the tangents positions at all points the minimal of them is chosen. Position of the tangent to the DRC at a point which corresponds to the minimum range (for example,

t_i) is assigned to the center of the range. Positions of the tangents in the previous and next point (t_{i-1}, t_{i+1}) is determined by taking into account the position of the tangent t_i.

The range of position of tangent t_{i+1} is limited by the tangent to the circle $ПК_{i+1}$ and line $(A;i+1)$. Line $(A;i+1)$ defined by passes through the point A which belongs to the tangent t_i so that the triangle $i;A;i+1$ is isosceles (ill. 2). Ranges of positions of tangents from t_{i-1} to t_1 and from t_{i+1} to t_n are defining similarly.

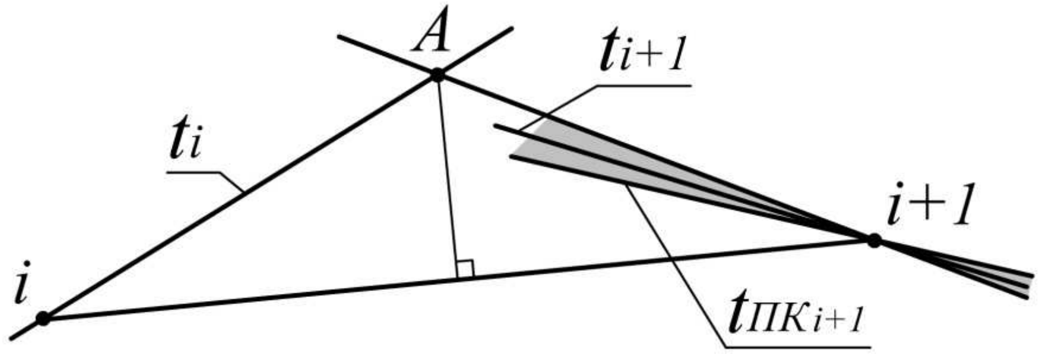

Illustration 2

In the result is a sequence of basic triangles, which allows modeling of contour with a monotonous change of curvature along the all points.

Basic triangles define ranges of the radiuses of curvature in the initial points, which can be provided for modeling a monotonous curve [3]. In order to be able to modeling the DRC with monotonous change of curvature along the all point, it is necessary that the ranges of curvature which determined by previous and next basic triangle were intersected. If these ranges do not intersect, it is necessary to make an adjustment of the positions of tangents in the initial points.

Adjusting of the tangent is performed using its rotation around the corresponding point of initial DRC. The range of radiuses of curvature at the respective point increases when turned clockwise. A boundary is a position of the tangent at which one of the basic triangles, the sides of which determines the tangent is isosceles.

Necessary condition for ensuring crossing the ranges of the radiuses of curvature at the point is the location of the initial points according to the conditions set out in [1].

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Conclusions and prospects of further researches

The following results were obtained in this work:

- the condition of the location of the tangents (normals) at the points that limit the segment of monotonous curve is defined. The criterion is ratio of sides of basic triangles or the location of the point of intersection of normals in initial points;

- the method of assigning of the positions of tangents (normals) at the points of curve in which the problem of modeling a contour with a monotonous change of curvature has a solution along the all points is suggested in this article.

The results obtained allow to model DRC along the points set which is composed of any number of points.

LITERATURE

1. Гавриленко Е.А. Дискретное интерполирование плоских одномерных обводов с закономерным изменением кривизны: дис. канд. техн. наук / Е.А. Гавриленко – Мелитополь, 2004. – 182 с.
2. Гавриленко Є.А. Визначення положення точки згущення при моделюванні монотонної дискретно представленої кривої / Є.А. Гавриленко// Прикл геом. та інж. графіка / Праці ТДАТУ – Вип.4, Т.53.–Мелітополь 2012.– с. 29-33.
3. Гавриленко Є.А. Визначення діапазонів геометричних характеристик монотонної дискретно представленої кривої / Є.А. Гавриленко, Ю.В. Холодняк // Прикл. геом. та інж. графіка / Праці ТДАТУ – Вип.4, Т.54. – Мелітополь 2012. – с. 38-42.
4. Рашевский П.К. Курс дифференциальной геометрии / П.К. Рашевский –М., ГИТТЛ, 1956.– 420 с.
5. Холодняк Ю.В. Формування ділянки дискретно представленої кривої із монотонною зміною кривини / Ю.В. Холодняк // Прикл. геом. та інж. графіка / Праці ТДАТУ – Вип.4, Т.57. – Мелітополь, 2013. – с. 211-216