THEORETICAL INVESTIGATION OF TURNING ABILITY OF MACHINE AND TRACTOR AGGREGATE ON BASIS OF PLOUGHING AND INTERTILLING WHEELED TRACTOR

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Abstract. The turning ability of tractors, which is estimated as a minimal value of the turning radius, is one of the most important characteristics of their efficient performance. As a result of theoretical simulation an equivalent scheme has been built for a complex of a wheeled tractor and a trailed agricultural machine, a system of equations has been built for uniform turning of the aggregate, and analytical expressions were defined for two unknown parameters: the curvature of the path of motion and the coordinates for displacement of the turning centre. As a result of numerical simulation graphic dependencies have been built: of the turning radius of the aggregate and the longitudinal coordinate on the turning angle of the steering wheels of the tractor; of the turning angles of the front and rear wheels of the tractor on the turning radius of its steering wheels; and of the turning angle of the machine and tractor aggregate on the turning angle of the steering wheels. The results of the theoretical research made it possible to estimate with high precision the turning ability of the machine and tractor aggregate depending on the design and kinematic parameters of the tractor and the trailed agricultural machine.

Keywords: wheeled tractor, machine and tractor aggregate, turning angle.

Introduction

A minimal turning radius of the tractor is an important kinematic characteristic of its performance, which is generally considered already when designing agricultural tractors. However, under the conditions of real operation of the wheeled tractors their turning radii may go beyond the estimated values, for instance, as a result of their operation within various machine and tractor aggregates. Therefore, the attempt to reduce the value of the turning radius of tractors used not as a separate unit but as a means of energy for a machine and tractor aggregate should be analytically considered and should have all-round substantiation.

General issues concerning investigations of the turning ability of the wheeled machine and tractor aggregates are reflected in quite a detailed way in a series of publications [1-4]. In other works [5-9] partial cases are treated with different approaches. In most cases these approaches contradict each other as to the distribution of the driving forces of the front and rear wheels of the aggregating tractor on the turning path in theoretical simulation of their turns. In the long run this leads to inaccuracies in analytical determination of the turning radii of tractors. Besides, not always the impact of the technological part of the machine and tractor aggregate upon the turning process of the tractor is considered.

Thus, a necessity arises to correct those methodological errors, which were committed in studying the turning ability of the machine and tractor aggregates on the basis of the ploughing and intertilling tractors, and to find analytical dependencies that provide a possibility to determine to a sufficient degree of accuracy the turning radii of aggregates made up of various trailed agricultural machines.

The aim of the present investigation is to build a mathematical model of an aggregate turning, allowing elaboration of a methodology for the estimation of the turning ability of a wide class of trailed machine and tractor aggregates used with the ploughing and intertilling tractors.

Materials and methods

In order to investigate the turning ability of a machine and tractor aggregate, let us make an estimation model. To simplify building of equations of the turning dynamics of a machine and tractor aggregate based on four-wheel drive tractors, we make the following initial assumptions.

1. Lateral interaction of the tractor tires with the bearing surface is described using the hypothesis of "the tire slip". Besides, the movement of the trailed part of the aggregate on the turning path is treated without considering the tire slip of its supporting running wheels;

2. The tangential forces of inertia of the tractor and the trailed technological part of the aggregate, due to their smallness, are not considered;
3. The difference between the angles of the tire slip of one geometrical axis is sufficiently small, and it can also be ignored;
4. The resistance coefficients of the tire slip of one geometrical axis are mutually equal.

Now let us build an equivalent scheme of turning of a machine and tractor aggregate presenting it as a wheeled ploughing and intertilling tractor, and a rear trailed agricultural machine making a turn (Fig. 1).

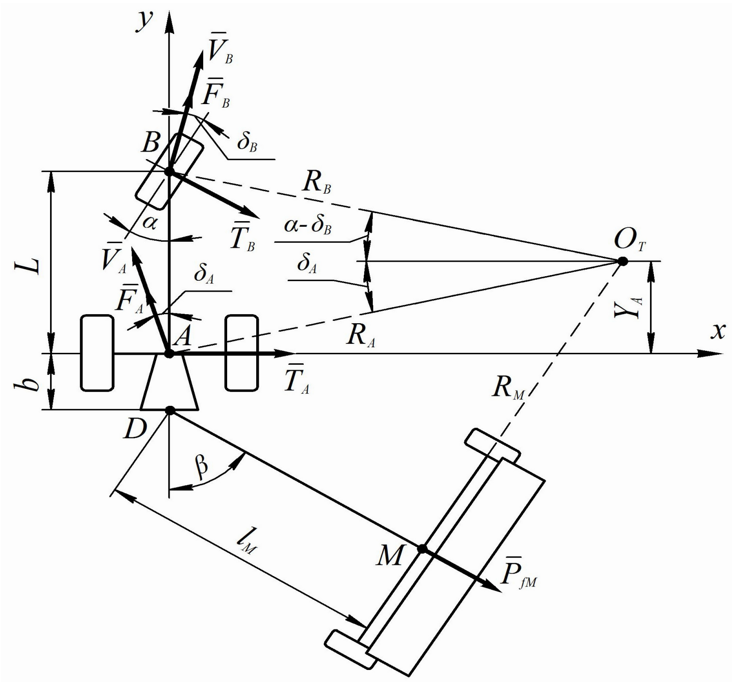

Fig. 1. **Equivalent scheme of turning of a machine and tractor aggregate on the basis of a four-wheel drive tractor during its turn**

Here, the front steerable axle of the tractor with two steerable wheels may be presented as one "equivalent" wheel. Let us designate the characteristic points in the scheme: A – the middle of the axle of the rear wheels, B – the centre of the equivalent wheel (the middle of the axle of the front steerable wheels), D – the connection point of the trailed agricultural machine, M – the middle of the aggregated agricultural machine. The longitudinal base of the tractor is designated by L, by b – the distance from the axis of the driving wheels of the tractor to the hitch point of the agricultural machine, l_M – the distance from the suspension centre of the agricultural machine to the axis of its supporting wheels (the length of the tow bar). The front steerable equivalent wheel is turned at the angle α, the tow bar of the agricultural machine is turned at the angle β.

As turning of the machine and tractor aggregate is carried out in one plane, we introduce into this plane a rectangular Cartesian system of coordinates xAy, placing the origin of coordinates at the point A and drawing the y axis in the direction of the longitudinal axis of the tractor towards the movement of the aggregate.

While turning of the given machine and tractor aggregate, a series of corresponding forces are acting on it. They are the driving force $\overline{F}_A$ of the rear and the driving force $\overline{F}_A$ of the front wheels of the tractor, applied at the points A and B, representing the middle points of the respective axes (axles) of the tractor. Besides, at the points A and B there are concentrated lateral forces $\overline{T}_A$ and $\overline{T}_B$, directed perpendicularly to the planes of the wheels of the respective axles of the tractor. While moving along the turning path, the impact of the trailed technological part of the aggregate upon the tractor is represented by the resistance force $\overline{P}_{fM}$ to rolling of the aggregated agricultural machine (See Fig. 1). Under the influence of external forces the wheels of the tractor roll over with a tire slip, the angles of which are δ_B for the front axis, and δ_A for the rear axis. Just these are the angles along the directions of which there are directed the above-mentioned driving forces $\overline{F}_A$ and $\overline{F}_B$. The directions of the vectors of these driving forces $\overline{F}_A$ and $\overline{F}_B$ coincide with the vectors of the corresponding velocities $\overline{V}_A$ and $\overline{V}_B$. Because of the tire slip phenomenon of a four-wheel drive tractor the instantaneous turning centre of the machine and tractor aggregate situated at the point O_T, at the crossing of perpendiculars drawn through the points A and B towards the vectors of velocities $\overline{V}_A$ and $\overline{V}_B$ (or, correspondingly, the forces $\overline{F}_A$ and $\overline{F}_B$), is displaced in a longitudinal direction to the value Y_A (See Fig. 1), which leads to the increase of its turning radius.

Results and discussion

It follows from what was laid out in the previous part that there are two unknown parameters for the discussed machine and tractor aggregate: the turning radius R_A, or else it is possible to use the curvature of the turning path K, which is defined by the following dependency: $K = 1/R_A$, and the displacement of the instantaneous turning centre Y_A. In order to determine them, it is sufficient to have a system of equilibrium equations. Such equations, which enter into the system, may be – the algebraic sum of the projections of all the forces onto Ay axis and the algebraic sum of the moments of all the forces in relation to the point O_T, which should be equal to zero, i.e.:

$$\left.\begin{aligned}&\sum_{k=1}^{n} P_{ky} = 0,\\ &\sum_{k=1}^{n} M_{O_T}(\overline{P}_k) = 0,\end{aligned}\right\} \tag{1}$$

Let us compose the first equation of the system (1):

$$F_B \cos\left(\alpha - \delta_B\right) - T_B \sin\alpha + F_A \cos\delta_A - P_{fM} \cos\beta = 0. \tag{2}$$

Considering the fact that the tire slip angles δ_B and δ_A of the tractor wheels are sufficiently small even when the tractor is turning, we can make such assumptions with regard to them:

$$\begin{gathered}\sin\delta_A \approx tg\delta_A \approx \delta_A,\\ \sin\delta_B \approx tg\delta_B \approx \delta_B,\\ \cos\delta_A = 1,\\ \cos\delta_B = 1.\end{gathered} \tag{3}$$

Taking into consideration expressions (3) equation (2) assumes the following form:

$$F_B \cos\left(\alpha - \delta_B\right) - T_B \sin\alpha + F_A - P_{fM} \cos\beta = 0. \tag{4}$$

Further, we apply transformations known from trigonometry which allow obtaining for $\cos(\alpha - \delta_B)$ such an expression:

$$\cos\left(\alpha - \delta_B\right) = \cos\alpha \cdot \cos\delta_B + \sin\alpha \cdot \sin\delta_B = \cos\alpha + \sin\alpha \cdot \delta_B. \tag{5}$$

After embedding expression (5) into expression (4) we obtain the first equilibrium equation in system (1) in the form:

$$F_B\left(\cos\alpha + \sin\alpha \cdot \delta_B\right) - T_B \sin\alpha + F_A - P_{fM} \cos\beta = 0. \tag{6}$$

Let us compose the second equation of system (1), which can be written as:

$$F_B R_B - T_B \sin\delta_B \cdot R_B - T_A Y_A + F_A R_A - P_{fM} R_M = 0. \tag{7}$$

Considering that the angle δ_B is small, equation (7) can be transformed into the form:

$$F_B R_B - T_B \delta_B R_B - T_A Y_A + F_A R_A - P_{fM} R_M = 0. \tag{8}$$

The lateral forces T_A and T_B in equations (6) and (8) can be expressed applying the dependencies known from the tractor and automobile theory:

$$T_A = k_A \delta_A , \tag{9}$$

$$T_B = k_B \delta_B , \tag{10}$$

where k_A and k_B – the tire slip resistance coefficients of the rear and front wheels of the tractor.

Further, let us determine the tire slip angles of the tractor.

According to the equivalent scheme (Fig. 1) it is possible to determine sinδ_A, which will be equal to:

$$\sin \delta_A = \frac{Y_A}{R_A} . \tag{11}$$

Since K is the curvature of the path and the angle δ_A has a very small value, expression (11) can be presented by such a dependency:

$$\delta_A = \sin \delta_A = Y_A K . \tag{12}$$

It is also evident from the equivalent scheme (Fig. 1) that the tire slip angle δ_B of the rear wheels of the tractor can be found from the dependency:

$$tg\left(\alpha - \delta_B\right) = \frac{\left(L - Y_A\right)}{R_A \cos \delta_A} , \tag{13}$$

where L – the longitudinal base of the tractor.

Taking into account that the angle δ_B has a minimal value and applying the fundamentals of trigonometry, we can write such an analytical expression:

$$tg\left(\alpha - \delta_B\right) = \frac{tg\alpha - tg\delta_B}{1 + tg\alpha \cdot tg\delta_B} . \tag{14}$$

Finally, the value of the tire slip angle δ_B of the front wheels of the tractor will be determined by means of the expression:

$$\delta_B = \frac{tg\alpha - \left(L - Y_A\right)K}{1 + tg\alpha\left(L - Y_A\right)K} . \tag{15}$$

Taking into consideration the obtained expressions (12) and (15), the lateral forces T_A and T_B can be determined by the following dependencies:

$$T_A = k_A Y_A K , \tag{16}$$

$$T_B = \frac{k_B\left[tg\alpha - \left(L - Y_A\right)K\right]}{\left[1 + tg\alpha\left(L - Y_A\right)K\right]} . \tag{17}$$

As it is evident from expressions (16) and (17), the values of the lateral forces T_A and T_B depend both on the curvature K of the motion path of the machine and tractor aggregate, and on the longitudinal displacement coordinate Y_A of the turning centre.

Finally, considering (9), (10), (16) and (17), the system of equilibrium equations (1) will assume the following appearance:

$$\left.\begin{aligned} &F_B\left(\cos\alpha + \sin\alpha \cdot \delta_B\right) - T_B \sin\alpha + F_A - P_{fM} \cos\beta = 0, \\ &F_B R_B - T_B \delta_B R_B - T_A Y_A + \frac{F_A}{K} - P_{fM} R_M = 0. \end{aligned}\right\} \tag{18}$$

where

$$T_A = k_A \delta_A = k_A Y_A K,$$
$$T_B = k_B \frac{\left[tg\alpha - \left(L - Y_A\right) K\right]}{\left[1 + tg\alpha\left(L - Y_A\right) K\right]},$$
$$\delta_B = \frac{\left[tg\alpha - \left(L - Y_A\right) K\right]}{\left[1 + tg\alpha\left(L - Y_A\right) K\right]},$$
$$R_B = \frac{1}{\left[K\left(\cos\alpha + \sin\alpha \cdot \delta_B\right)\right]},$$
$$R_M = \frac{\left(Y_A + b + l_M \cos\beta\right)}{\sin\beta},$$
$$l_M = R_A\left(\sin\beta - \cos\beta \cdot \delta_A\right) - b\cos\beta.$$

Consequently, just the system of equations (18) represents an estimation mathematical model of turning of the machine and tractor aggregate on the basis of a four-wheel drive tractor.

The resistance force $\overline{P}_{fM}$ to rolling of the aggregated agricultural machine is found from the following well-known expression:

$$P_{fM} = M_M g \cdot f_k, \tag{19}$$

where M_M – the mass of the trailed agricultural machine;
g – acceleration of gravity;
f_k – coefficient of the rolling resistance.

For estimation of the draft force F_A of the rear axle and the draft force F_B of the front axle of a four-wheel drive tractor, one can use with a sufficient degree of accuracy the following dependencies:

$$F_A = G_A f_k + \frac{1}{2} P_{fM}, \tag{20}$$

$$F_B = G_B f_k + \frac{1}{2} P_{fM}, \tag{21}$$

where G_A, G_B – the summary forces of gravity applied, respectively, to the rear and the front axles of a ploughing and intertilling tractor.

In addition, to make further calculations, we determine the summary forces G_A and G_B applied, respectively, to the rear and the front axles of a wheeled tractor.

At the point A there is concentrated the force of gravity G_A, which can be determined in such a way:

$$G_A = G_{EB} + G_{MB}, \tag{22}$$

where G_{EB} – the force of the operating weight of the tractor applied to its rear axle without the weight of the aggregated agricultural machine;
G_{MB} – force of the weight of the rear-mounted agricultural machine attached to the rear axle of the tractor.

At the other point B there is concentrated the force of gravity G_B, which can be found from this expression:

$$G_B = G_{EF} + G_{MF}, \tag{23}$$

where G_{EF} – the force of the operating weight of the tractor applied to its front axle without the weight of the front-mounted agricultural machine;

G_{MF} – force of the weight of the front-mounted agricultural machine (for a case when an agricultural machine or any other tool is mounted in front of the tractor).

It is necessary to take into account the fact that the summary forces of gravity (G_A and G_B) on each of the axles of the tractor must not exceed the allowed vertical loads on the two rear and two front tires of its wheels. In a particular case estimation was made for a wheeled tractor, the drawbar category 30 kN, equipped with tires of the type 16.9R38, with the internal air pressure ρ_{pt} = 0.2 MPa, according to the following parameters: the allowed vertical loads – 27 kN; the diameter of the tire– 1.68 m; the width of the tire – 0.43 m.

In order to determine the range of variations of the coefficients k_A and k_B of the tire spin resistance of a wheeled tractor, we use the well-known expression of R. Smiley and V. Horn [10]:

$$k = 60\left[1.75\frac{h_{pt}}{D_{pt}} - 12.7\left(\frac{h_{pt}}{D_{pt}}\right)^2\right]\rho_{pt}\cdot b_{pt}^2, \tag{24}$$

where h_{pt} – the depth of the track formed by the tire of the wheel;
D_{pt} – diameter of the tire of the wheel;
b_{pt} – width of the tire of the wheel.

As calculations, made according to the given formula, indicate, for each tire of the wheeled tractor the relation h_{pt}/D_{pt} has to be less than 0.0885.

Concrete values of the coefficients k_A and k_B of the tire spin resistance of the rear and front wheels of the tractor may be determined in our case by such an expression:

$$k_A\left(k_B\right) = 120\left[1,75\frac{h_{pt}}{D_{pt}} - 12.7\left(\frac{h_{pt}}{D_{pt}}\right)^2\right]\rho_{pt}\cdot b_{pt}^2\,. \tag{25}$$

Using the last expression, we find that changing the pressure of air ρ in the tires of the running wheels of the tractor of the particular class from 0.10 MPa to 0.13 MPa, the coefficients k_A and k_B of the tire spin resistance vary within the limits 130…170 kN·rad^{-1}. In our calculations we will change the parameter b in the interval 1.0...2.0 m, the length of the tow bar of the trailed agricultural machine l_M – within the limits 3…5 m, and the turning angle α of the steerable wheels – in the range 0.26...0.44 rad. (i.e., from 15° to 25°). The numerical calculations of the system of equations (18), carried out by means of a programme composed on the PC, showed that more intense diminishing of the turning radius R_A of the machine and tractor aggregate takes place when the steerable wheels of the tractor turn at the angle α, which is equal to approximately 20^o. If the impact upon the steerable wheels continues, i.e. the angle α is increased to 25°, this process becomes considerably slower (Fig. 2a).

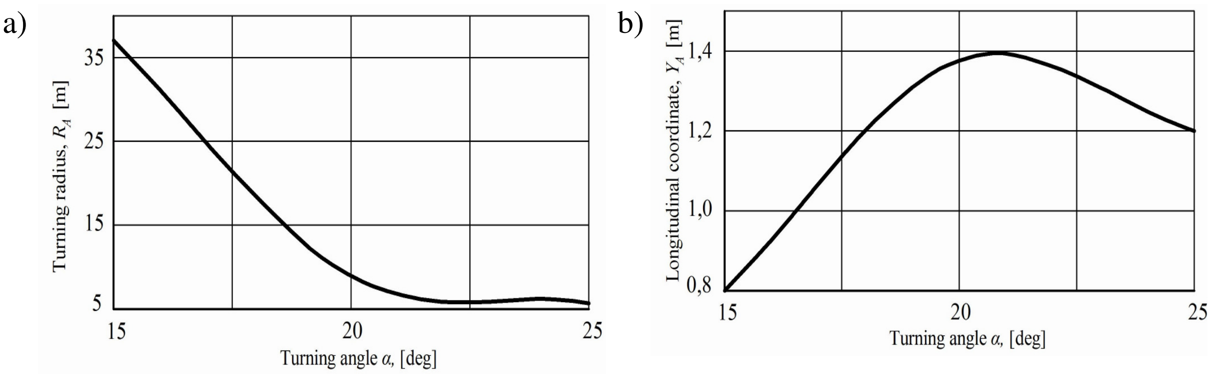

Fig. 2. **Dependence of the radius R_A (a) and the longitudinal coordinate Y_A (b) on the turning angle α of the steerable wheels of the tractor**

Such a behaviour of the turning radius R_A can be explained by the fact that increasing the angle α from 15° to 20°, the coordinate Y_A of the longitudinal displacement of the turning centre of the machine and tractor aggregate increases considerably intensely (Fig. 2b), as well as the wheel spin angle δ_A of the rear tires of the tractor (Fig. 3). The behaviour of the angle δ_B is opposite: being maximum (0.3 rad) at α = 15º, it gradually diminishes to 0.2 rad. at α = 20º (Fig. 3).

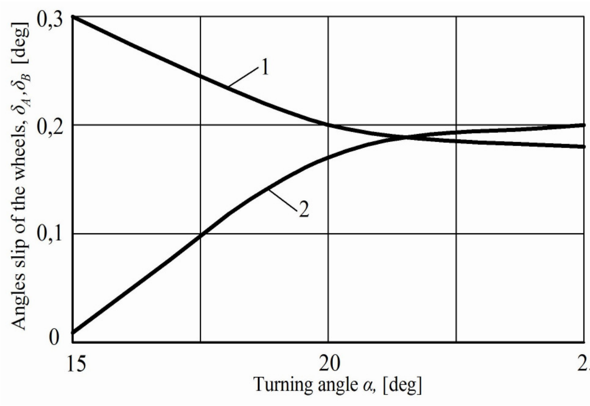

Fig. 3. **Dependence of the tire spin angles δ_A and δ_B of the front (1) and the rear (2) wheels of the tractor upon the turning angles α of its steerable wheels**

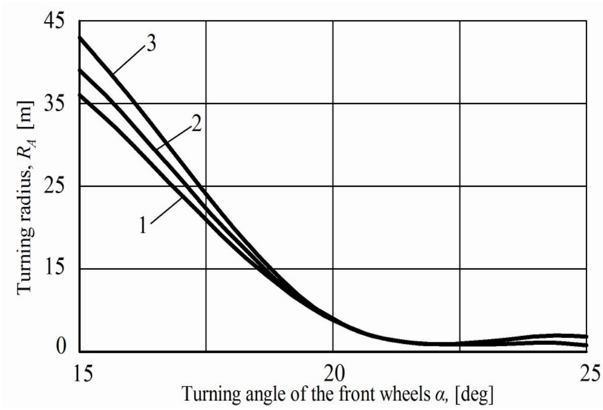

Fig. 4. **Dependence of the turning angle upon the turning angle of the steerable wheels of the tractor at different lengths of the tow bar:** 1 – l_M = 3 m; 2 – l_M = 4 m; 3 – l_M = 5 m

On the whole, just such processes of changing the parameters Y_A and the wheel spin angles δ_A and δ_B of the wheels of the ploughing and intertilling tractors determine intense diminishing of the turning radius R_A of the machine and tractor aggregate within the range of a steerable impact α = 15…20º.

When the angle α continues increasing from 20º to 25º, the parameter Y_A decreases, although if only by 0.2 m (from 1.4 m to 1.2 m). The wheel spin angle δ_B of the front wheels of the tractor continues diminishing slowly (curve 1, Fig. 3), but the wheel spin angle δ_A of the rear wheels continues growing slowly (curve 2, Fig. 3). As a result of this, the turning radius R_A of the machine and tractor aggregate continues decreasing, though slowly (Fig. 2*a*).

It should be underlined that the process of changing $R_A = f(\alpha)$ is practically invariant in relation to the increase in the distance *b* from the axis of the rear wheels of the tractor to the hitch point *D* of the trailed agricultural machine from 1 m to 2 m. The difference between the values of the turning radius R_A of the machine and tractor aggregate in different variants *b* is not greater than 0.2 m. On most wheeled ploughing and intertilling tractors of class 3 the parameter *b* constitutes about 1.05 m. To install the parameter *b* less than 1 m is not only undesirable but impossible since at full turn of the steerable wheels of the ploughing and intertilling tractor it is quite possible that there may arise an impact contact of the tow bar of the aggregated tool with one of the rear wheels of the means of energy during the turns. But it is also as undesirable to increase the parameter *b* more than 1 m because of a significant increase in the kinematic length of the entire machine and tractor aggregate. Under conditions of operation of such a machine and tractor aggregate this may cause certain undesirable increase in the unproductive waste of the working time. It has been established by numerical calculations that the length of the tow bar l_M of the aggregated agricultural machine, which varies within the limits 3...5 m, does not affect either the changing process of the turning radius R_A of the aggregate under the steerable impact. The most noticeable this impact is only at the turning angle of the steerable wheels α = 15º (Fig. 4). In this case increasing the parameter l_M leads to a corresponding increase in the radius R_A. For instance, increasing the length of the tow bar l_M of the trailed agricultural machine from 3 m to 5 m, the turning radius R_A of the machine and tractor aggregate increases from 36 m to 43 m, that is, almost by 20 %. As the graphs presented in Fig. 4 witness, in order to increase the turning radius R_A of the machine and tractor aggregate on the basis of a four-wheel drive tractor, preference should be given to a trailed agricultural machine with less length l_M of its tow bar.

Conclusions

1. With the help of the newly composed equations for a uniform movement on the turning path of the machine and tractor aggregate on the basis of a four-wheel drive tractor regularities have been established for impact of the design parameters of a trailed agricultural machine and the tractor itself upon the indicators of its turning ability.

2. As a result of numerical simulation of the turning process of a machine and tractor aggregate, using an estimation mathematical model composed on the PC, the design parameters have been established, which ensure better turning ability of the aggregate. The most intense diminishing of the turning radius R_A of the particular machine and tractor aggregate takes place at the turns of the steerable tractor wheels up to the angle α equal to 20º.

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